A Review of FAME Spin Dynamics and the Effects of a Fluctuating Solar Irradiance

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- Conjecture:

  stochastic = "bad"

- Corollary:

  not stochastic = not "bad"

- Far-reaching consequences
  - Periodic and quasiperiodic physical processes can be modeled and hence reliably removed from the observational data
  - Stochastic perturbations are inherently not easily-modelable

A Few Spin Dynamics Issues

- **Variability of solar "constant"**
  - variation ~ 0.1 percent
  - T ~ days
  - some evidence for variations on the order of minutes
- Earth radiation pressure
  - visible
  - infrared
  - variability due to weather
  - complicated torques
    - spacecraft not protected by shield
    - optical ports
- Fuel sloshing
- Shield & flattop albedos
  - variable over time as materials age
  - spatial inhomogeneities
- "Ice skater" effect (eclipses)?
- Circulation of Sun around rotating frame stationary point
- Variations in shield angle
  - nonuniform in circumference
  - slow variation over time?
  - fast variation — flapping?
- Axis of shield misaligned with spacecraft spin axis
- Geotail particle bursts
  - "wind" gusts?
  - potentials across spacecraft surfaces $\rightarrow$ currents $\rightarrow$ magnetic torques
    - caused Echo spinup
- Variation of solar radiation pressure as spacecraft orbits the Earth
- Gravity gradient spin modulation
- Magnetic torques
- Magnetopause crossings
  - rare
  - short duration (~15 min) exposure to solar wind

Outline

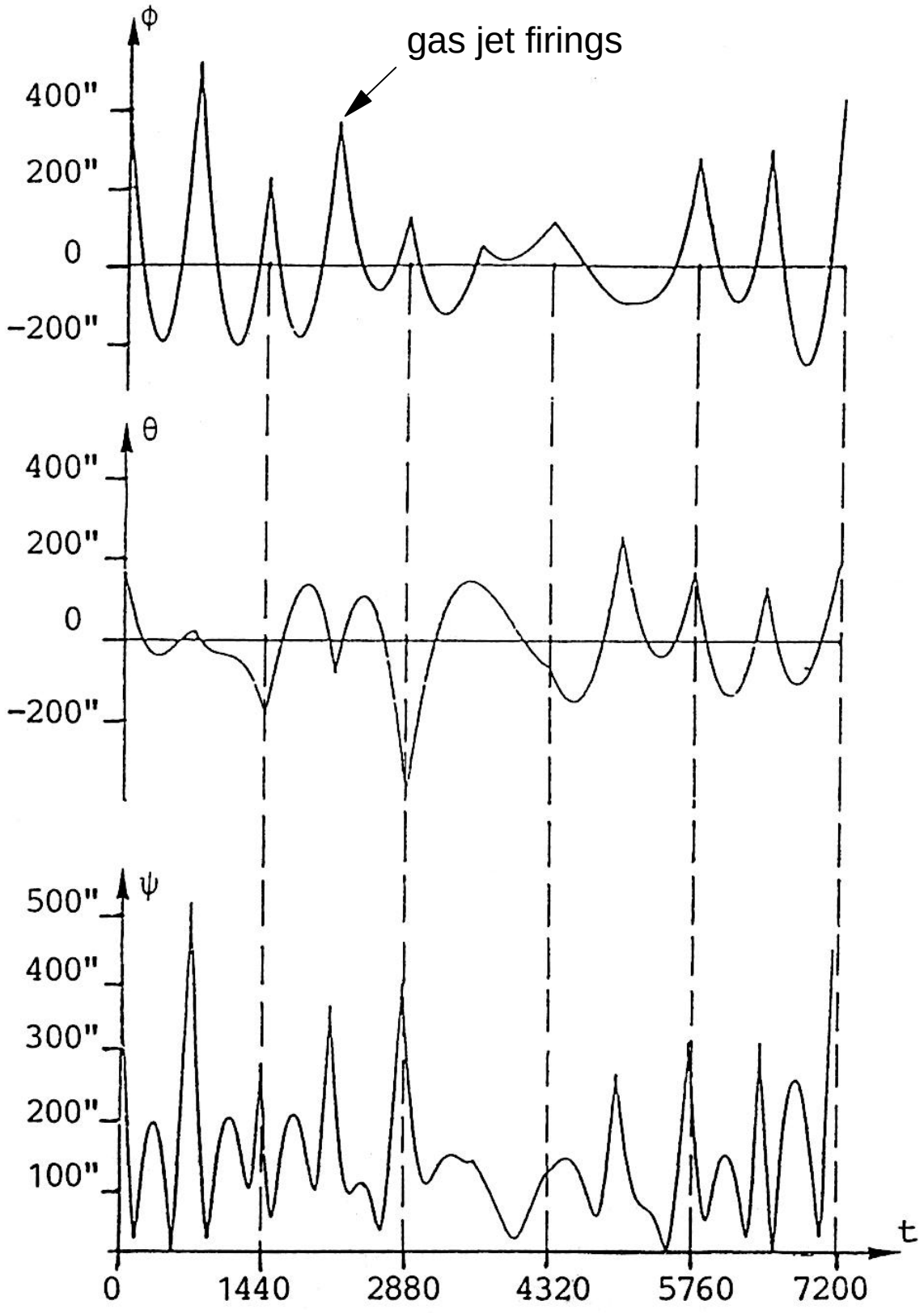
- Introduction
- Some mention of relevant spin dynamics
- Torque model development
- Basic behavior
- Solar irradiance variations
- Solar irradiance fluctuation effects on spacecraft attitude
- Simulated observations and measurement accuracy

Introduction

Introduction

- FAME satellite's precession will be driven by solar radiation pressure on the spacecraft's sun shield
- Advantage: continuity of data, resulting in increased mission accuracy
  - SAO group covariance studies (FTM99-05)
    - gain a factor of 4 in rotation coherence in going from 6 thruster firings per rotation to one firing per rotation
    - asymptotic limit: one order of magnitude accuracy gain
- Potential problem: irradiance fluctuations
  - Stochastic, therefore potentially unmodelable
- How can we reduce the impact from stochastic perturbations?
  - spin faster
  - reduce shield size
  - increase s/c mass or significantly alter the s/c mass distribution
- Key Question: What is the character and magnitude of the effects of solar irradiance fluctuations on the spacecraft spin dynamics, and hence the effects on measurement accuracy?

Hipparcos Attitude Corrections



Introduction (continued)

- What answering the Key Question entails
  - Understanding of rigid body dynamics
  - Development of a useful torque model
    - Pressure field model(s)
      - Solar wind (not a problem in geosynchronous orbit)
      - Solar radiation pressure — three approaches:
        - constant magnitude (bad approx.)
        - model the fluctuations (*very* difficult solar physics problem)
        - incorporate observational data (best way to go)
      - Earth radiation pressure (not considered in this study)
    - Spacecraft solar shield model
      - Smooth "skirt", swept back by an adjustable angle to control the precession rate (Reasenberg 1997, FTM97-05)
      - Fully analytic exact solution for the torque on such a shield (Murison 1998, DDA Charlottesville, http://aa.usno.navy.mil/murison/talks/)
  - Numerical program: exploration tool
  - Determine the effects on spacecraft attitude and measurement accuracy

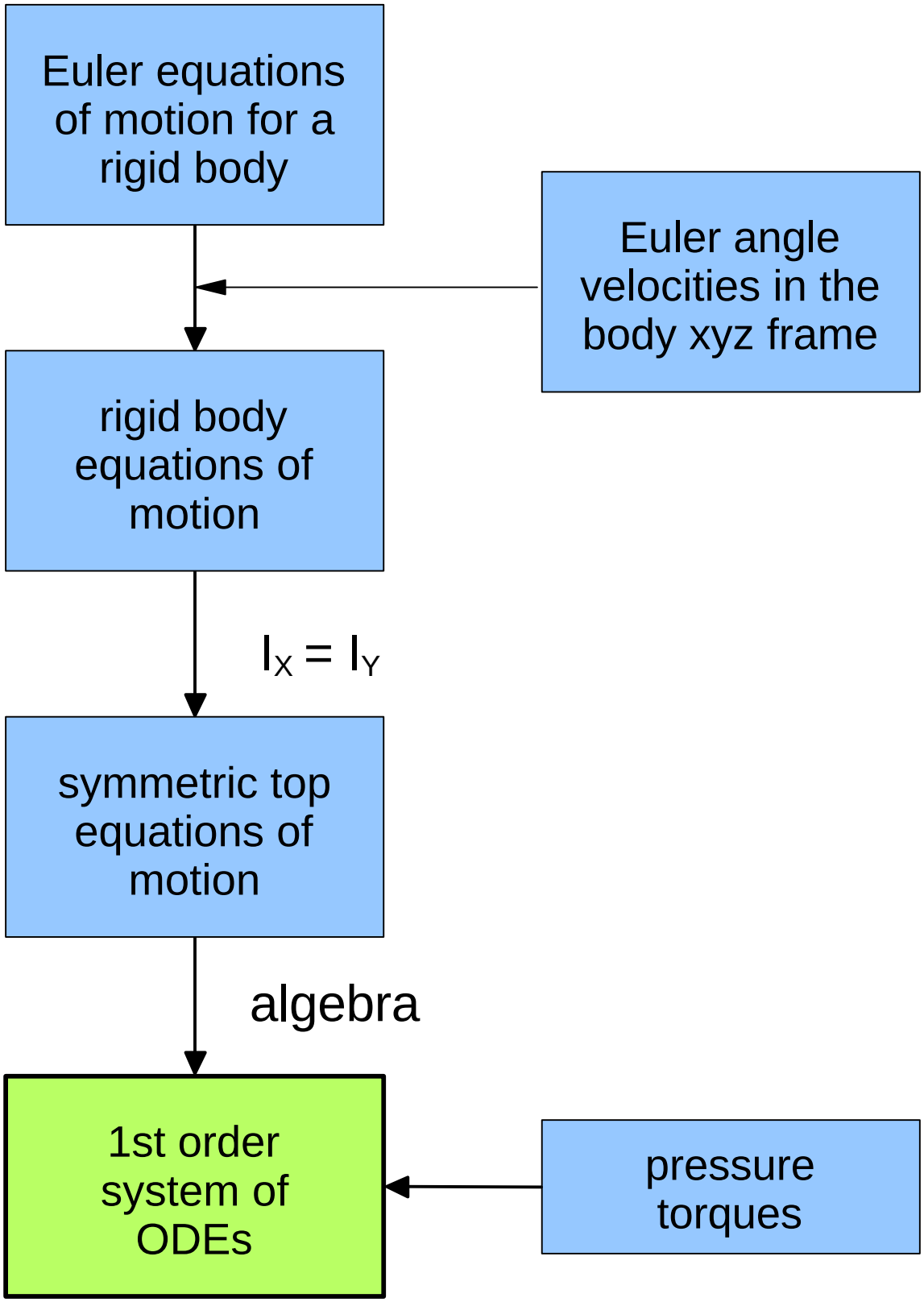
Some Mention of Spin Dynamics

Spacecraft Spin Dynamics

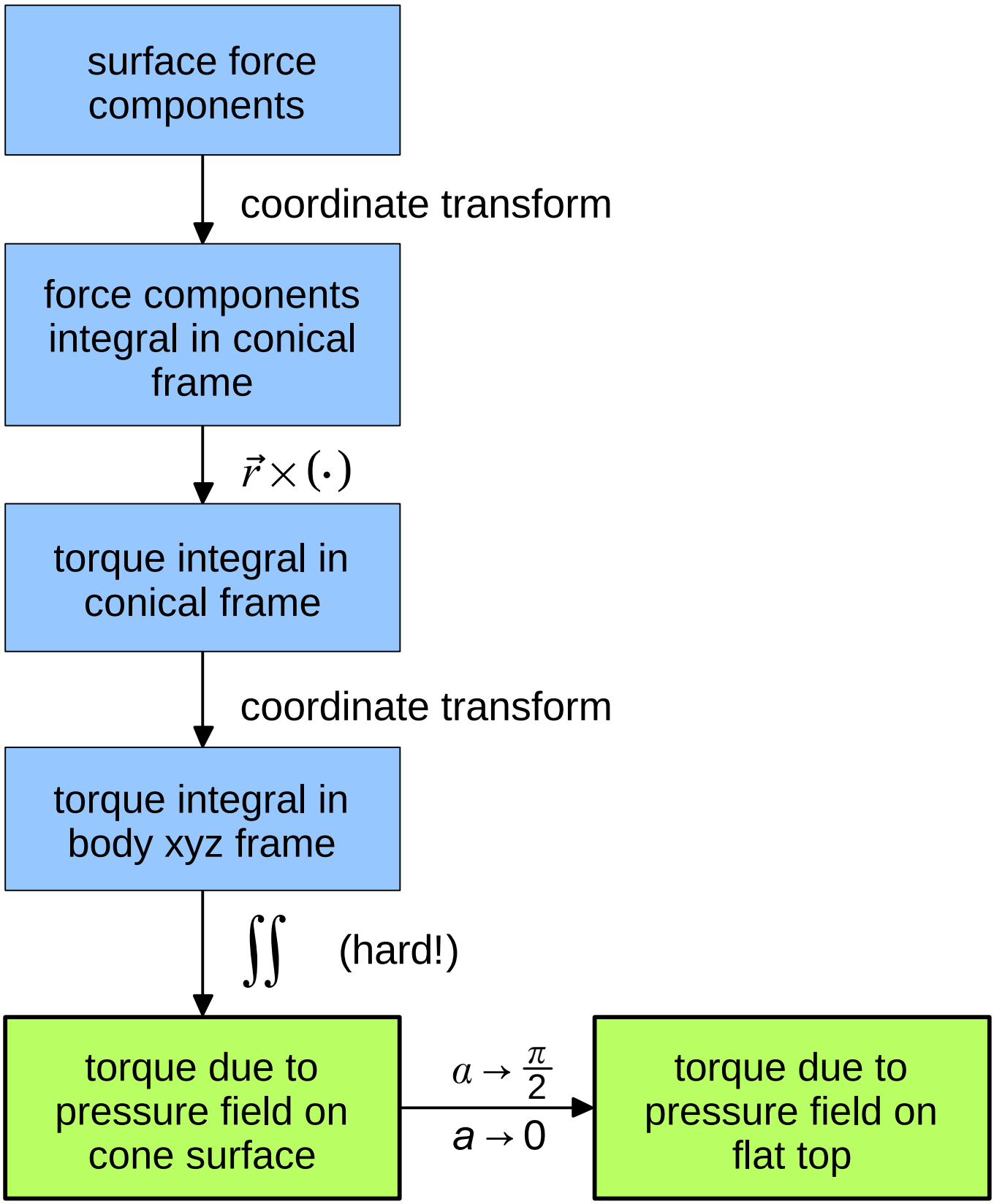
- Start with symmetric top equations of motion
- Full dynamical problem: guiding center motion around the sun direction
  - Reasenberg (1999) and V. Slabinsky (1976) (now at USNO) both independently discovered this solution
  - Simplifications for this particular study:
    - Fix the Sun in place and ignore Earth's orbital motion
    - Ignore s/c orbit around Earth
      - Earth and lunar perturbations
      - eclipses
      - gravity gradient torques
      - etc., etc., etc.

    ➡ ignore all smooth (and therefore modelable, and therefore removable from the data) perturbations
- Simplified problem: spinning, symmetric top, with attached conical shield, embedded in a radiation pressure field
  - Ignore solar wind (magnetosphere protection)
  - Integrate pressure field over cylinder "top" and solar shield to get torques

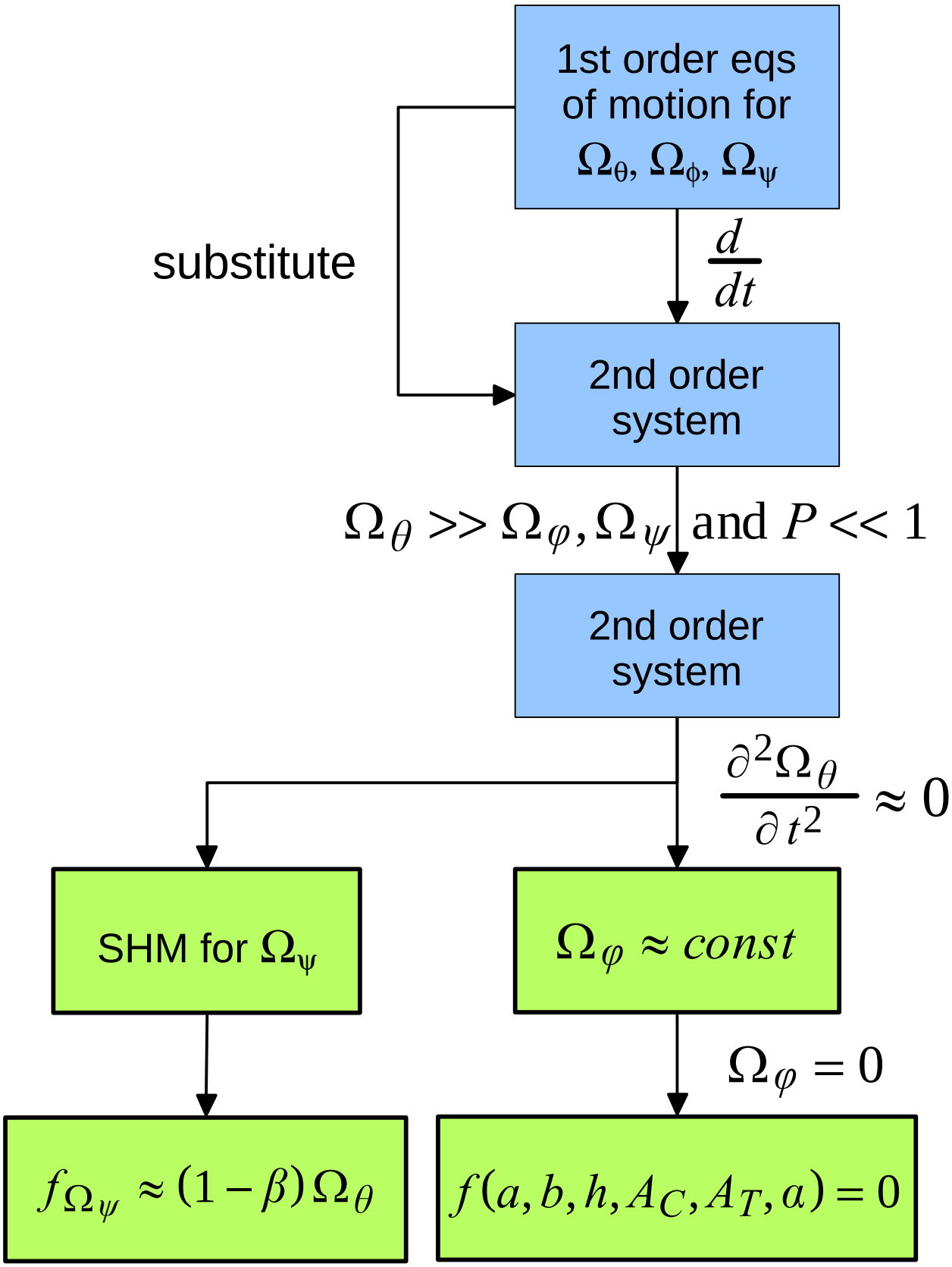
1. Equations of Motion



2. Torque Calculations (Cone & Top)



3. Precession Calculation



Euler Angles

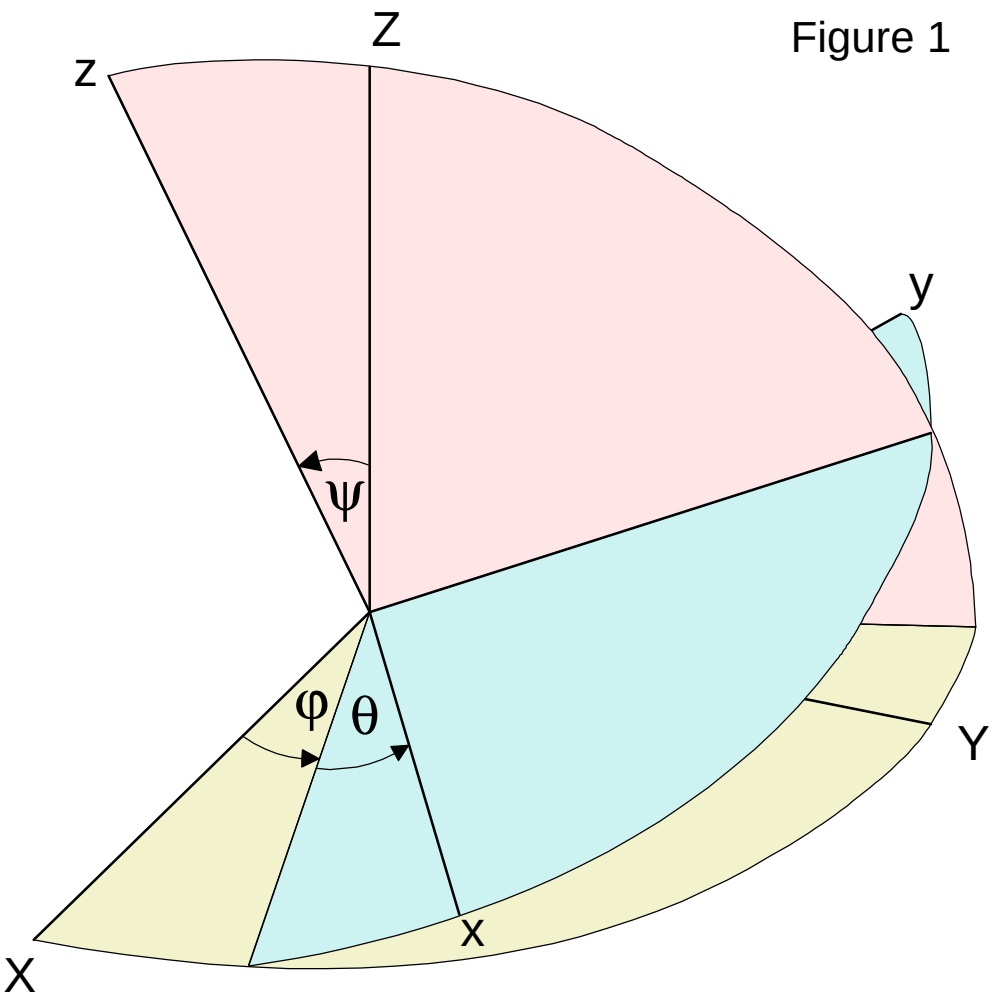


Figure 1

Equations of Motion

- Start with the Euler equations for a rigid body:

$$\left.\begin{aligned} I_x \frac{d}{dt}\Omega_x + (I_z - I_y)\,\Omega_y \Omega_z - K_x = 0 \\ I_y \frac{d}{dt}\Omega_y + (I_x - I_z)\,\Omega_x \Omega_z - K_y = 0 \\ I_z \frac{d}{dt}\Omega_z + (I_y - I_x)\,\Omega_x \Omega_y - K_z = 0 \end{aligned}\right\}$$

- Write the components of W along the rotation axes as projections onto the body xyz frame:

$$\vec{\Omega}_\varphi + \vec{\Omega}_\psi + \vec{\Omega}_\theta = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} = \begin{bmatrix} \frac{d\varphi}{dt}\sin\theta\,\sin\psi + \frac{d\psi}{dt}\cos\theta \\ \frac{d\varphi}{dt}\cos\theta\,\sin\psi - \frac{d\psi}{dt}\sin\theta \\ \frac{d\varphi}{dt}\cos\psi + \frac{d\theta}{dt} \end{bmatrix}$$

- Substitute into the Euler equations to get

$$\frac{d^2\psi}{dt^2}\cos\theta + \frac{d^2\varphi}{dt^2}\sin\theta\,\sin\psi + \left[\frac{I_z - I_y}{I_x}\left(\frac{d\varphi}{dt}\right)^2\cos\psi + \frac{I_x - I_y + I_z}{I_x}\frac{d\theta}{dt}\frac{d\varphi}{dt}\right]\sin\psi\,\cos\theta$$
$$+\left(\frac{I_x + I_y - I_z}{I_x}\frac{d\varphi}{dt}\cos\psi - \frac{I_x - I_y + I_z}{I_x}\frac{d\theta}{dt}\right)\frac{d\psi}{dt}\sin\theta - \frac{K_x}{I_x} = 0$$

$$-\frac{d^2\psi}{dt^2}\sin\theta + \frac{d^2\varphi}{dt^2}\cos\theta\,\sin\psi + \left[\frac{I_x - I_z}{I_y}\left(\frac{d\varphi}{dt}\right)^2\cos\psi + \frac{I_x - I_y - I_z}{I_y}\frac{d\theta}{dt}\frac{d\varphi}{dt}\right]\sin\psi\,\sin\theta$$
$$+\left(\frac{I_x + I_y - I_z}{I_y}\frac{d\varphi}{dt}\cos\psi + \frac{I_x - I_y - I_z}{I_y}\frac{d\theta}{dt}\right)\frac{d\psi}{dt}\cos\theta - \frac{K_y}{I_y} = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{d^2\varphi}{dt^2}\cos\psi - \frac{I_x - I_y}{I_z}\left(\frac{d\varphi}{dt}\right)^2\cos\theta\,\sin\theta\,\sin^2\psi$$
$$+\left(2\frac{I_x - I_y}{I_z}\sin^2\theta - \frac{I_x - I_y + I_z}{I_z}\right)\frac{d\psi}{dt}\frac{d\varphi}{dt}\sin\psi + \frac{I_x - I_y}{I_z}\left(\frac{d\psi}{dt}\right)^2\cos\theta\,\sin\theta - \frac{K_z}{I_z} = 0$$

Symmetric Top

- We have a "symmetric top", so let $I_x=I_y=I_{xy}$ and

$$\beta \equiv \frac{I_{xy}-I_z}{I_{xy}}$$

We find

$$\frac{d^2\psi}{dt^2}\cos\theta+\frac{d^2\varphi}{dt^2}\sin\theta\sin\psi+\left[(1-\beta)\frac{d\theta}{dt}\frac{d\varphi}{dt}-\beta\left(\frac{d\varphi}{dt}\right)^2\cos\psi\right]\sin\psi\cos\theta$$
$$+\left[(1+\beta)\frac{d\varphi}{dt}\cos\psi-(1-\beta)\frac{d\theta}{dt}\right]\frac{d\psi}{dt}\sin\theta-\frac{K_x}{I_{xy}}=0$$

$$-\frac{d^2\psi}{dt^2}\sin\theta+\frac{d^2\varphi}{dt^2}\cos\theta\sin\psi-\left[(1-\beta)\frac{d\theta}{dt}\frac{d\varphi}{dt}-\beta\left(\frac{d\varphi}{dt}\right)^2\cos\psi\right]\sin\psi\sin\theta$$
$$+\left[(1+\beta)\frac{d\varphi}{dt}\cos\psi-(1-\beta)\frac{d\theta}{dt}\right]\frac{d\psi}{dt}\cos\theta-\frac{K_y}{I_{xy}}=0$$

$$\frac{d^2\theta}{dt^2}+\frac{d^2\varphi}{dt^2}\cos\psi-\frac{d\varphi}{dt}\frac{d\psi}{dt}\sin\psi-\frac{K_z}{(1-\beta)I_{xy}}=0$$

- The third equation is conservation of angular momentum along the symmetry axis:

$$\frac{d}{dt}\left(\frac{d\theta}{dt}+\frac{d\varphi}{dt}\cos\psi\right)=\frac{K_z}{(1-\beta)I_{xy}}$$

Symmetric Top (cont.)

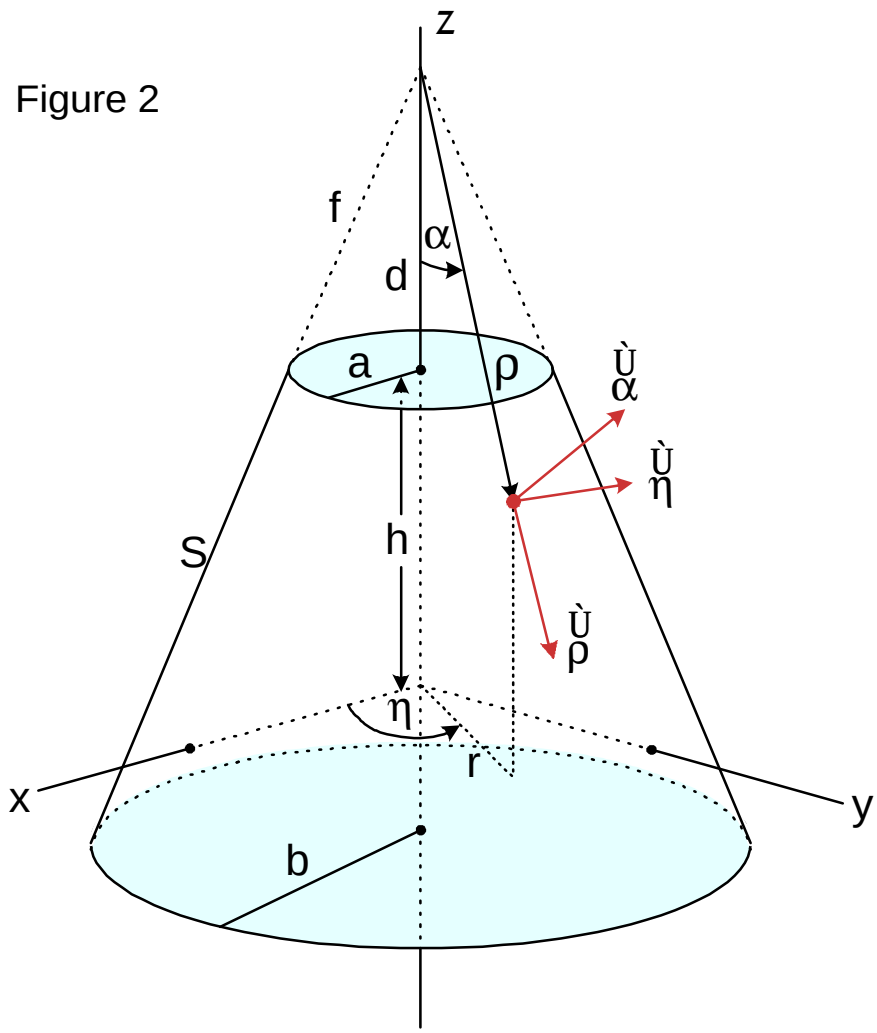
- Convert to a system of first-order ODEs:

$$
\begin{aligned}
\frac{d\varphi}{dt} &= \Omega_\varphi \\
\frac{d\psi}{dt} &= \Omega_\psi \\
\frac{d\theta}{dt} &= \Omega_\theta \\
\sin\psi \frac{d}{dt}\Omega_\varphi &= [(1-\beta)\Omega_\theta - (1+\beta)\cos\psi\,\Omega_\varphi]\Omega_\psi + \frac{K_x \sin\theta + K_y \cos\theta}{I_{xy}} \\
\frac{d}{dt}\Omega_\psi &= \left[\beta\cos\psi\,\Omega_\varphi^2 - (1-\beta)\Omega_\theta\Omega_\varphi\right]\sin\psi + \frac{K_x\cos\theta - K_y\sin\theta}{I_{xy}} \\
\sin\psi \frac{d}{dt}\Omega_\theta &= [(1+\beta\cos^2\psi)\Omega_\varphi - (1-\beta)\cos\psi\,\Omega_\theta]\Omega_\psi \\
&\quad + \frac{K_z \sin\psi}{(1-\beta)I_{xy}} - \frac{K_x\sin\theta + K_y\cos\theta}{I_{xy}}\cos\psi
\end{aligned}
$$

Torque Model Development

Conical Coordinate Frame

Figure 2



Force Components on Surface Element

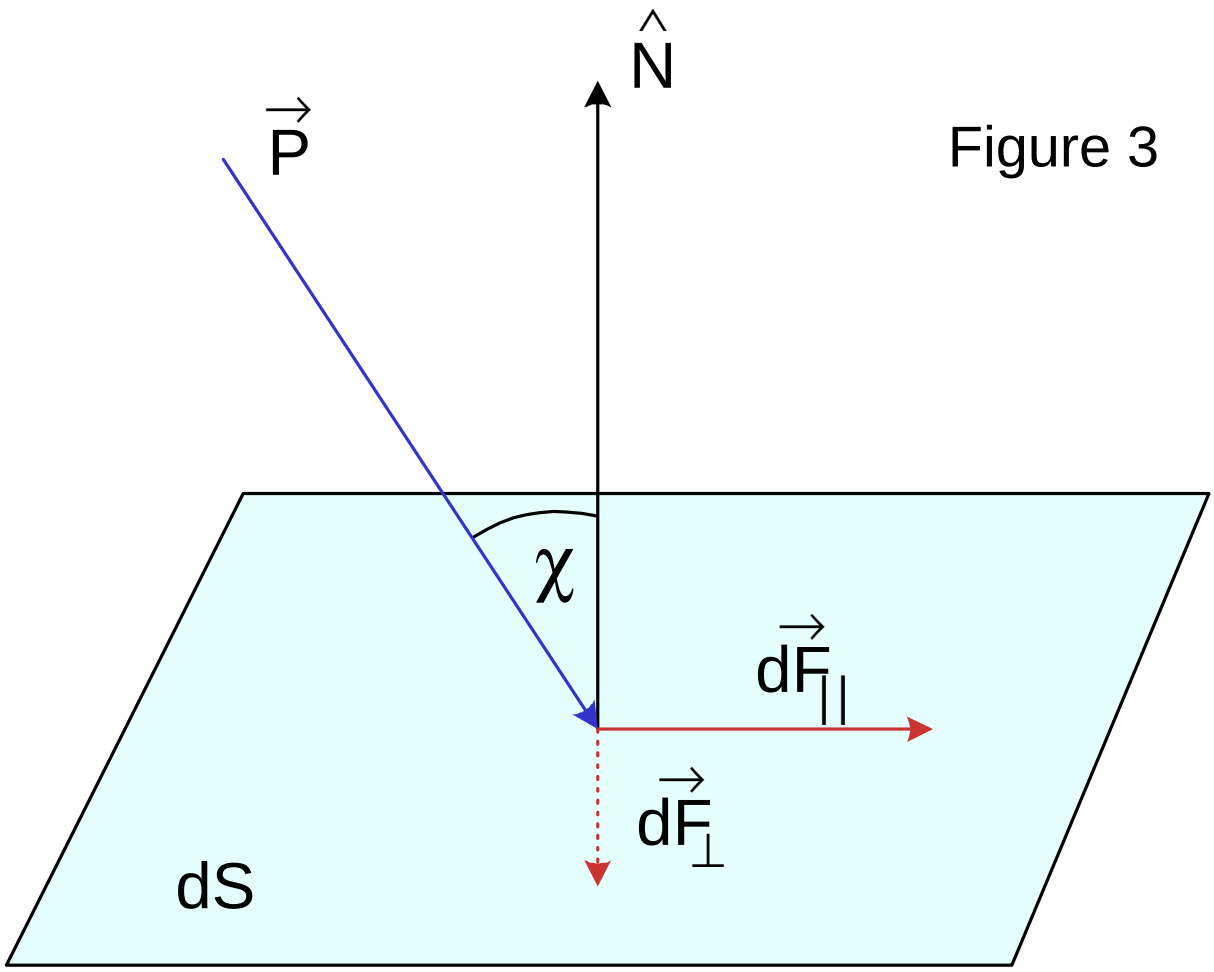


Figure 3

Force Due to Radiation Pressure

- "Conical" coordinates:

$$
\begin{aligned}
x &= \rho \sin\alpha \cos\eta \\
y &= \rho \sin\alpha \sin\eta \\
z &= h + \frac{a}{\tan\alpha} - \rho \cos\alpha
\end{aligned}
$$

- Infinitesimal force components perpendicular to and tangent to the cone surface:

$$
\begin{bmatrix} dF_{\perp} \\ dF_{\parallel} \end{bmatrix} = P \cdot d\Sigma \cdot |\cos\gamma| \cdot \begin{bmatrix} (1+A)\cos\gamma \\ (1-A)\sin\gamma \end{bmatrix}
$$

- A short calculation reveals that

$$
\begin{bmatrix} dF_{\rho} \\ dF_{\eta} \\ dF_{\alpha} \end{bmatrix} = P \cdot d\Sigma \cdot \cos\chi \cdot \begin{bmatrix} (1-A)\pi_{\rho} \\ (1-A)\pi_{\eta} \\ (1-A)\pi_{\alpha} - 2A\cos\chi \end{bmatrix}
$$

where

$$
\cos\chi \equiv -(\hat{P} \cdot \hat{N}) = -\cos\gamma
$$

$$
\pi_{\rho} \equiv \frac{P_{\rho}}{P} \qquad \pi_{\eta} \equiv \frac{P_{\eta}}{P} \qquad \pi_{\alpha} \equiv \frac{P_{\alpha}}{P}
$$

Torque Due to Radiation Pressure

- From $\begin{bmatrix} \pi_\rho \\ \pi_\eta \\ \pi_\alpha \end{bmatrix} = \mathbb{R}(\alpha, \eta)^{-1}\, \mathbb{R}(\varphi, \psi, \theta) \begin{bmatrix} \pi_X \\ \pi_Y \\ \pi_Z \end{bmatrix}$

  we have (third component)

$$\begin{aligned}
\cos\chi \;=\; -\pi_\alpha \;=\; & -\{\cos\alpha\,[\cos\eta\,(\cos\theta\,\cos\varphi - \sin\theta\,\cos\psi\,\sin\varphi) \\
& \qquad - \sin\eta\,(\sin\theta\,\cos\varphi + \cos\theta\,\cos\psi\,\sin\varphi)] + \sin\alpha\,\sin\psi\,\sin\varphi\}\,\pi_X \\
& -\{\cos\alpha\,[\cos\eta\,(\cos\theta\,\sin\varphi + \sin\theta\,\cos\psi\,\cos\varphi) \\
& \qquad - \sin\eta\,(\sin\theta\,\sin\varphi - \cos\theta\,\cos\psi\,\cos\varphi)] - \sin\alpha\,\sin\psi\,\cos\varphi\}\,\pi_Y \\
& -[\cos\alpha\,(\cos\eta\,\sin\theta\,\sin\psi + \sin\eta\,\cos\theta\,\sin\psi) + \sin\alpha\,\cos\psi]\,\pi_Z
\end{aligned}$$

$$\pi_X \equiv \frac{P_X}{P} \qquad \pi_Y \equiv \frac{P_Y}{P} \qquad \pi_Z \equiv \frac{P_Z}{P}$$

- We can integrate over the conical surface:

$$\begin{bmatrix} F_\rho \\ F_\eta \\ F_\alpha \end{bmatrix} = P \int_0^{2\pi} \int_f^{f+S} \cos\chi \cdot \begin{bmatrix} (1 - A_C)\,\pi_\rho \\ (1 - A_C)\,\pi_\eta \\ (1 - A_C)\,\pi_\alpha - 2\,A_C \cos\chi \end{bmatrix} \cdot \rho \sin\alpha \; d\rho \; d\eta$$

- The torque is then

$$\begin{bmatrix} K_\rho \\ K_\eta \\ K_\alpha \end{bmatrix} = \int_0^{2\pi} \int_f^{f+S} \vec{r} \times \begin{bmatrix} (1 - A_C)\,\pi_\rho \\ (1 - A_C)\,\pi_\eta \\ (1 - A_C)\,\pi_\alpha - 2\,A_C \cos\chi \end{bmatrix} \cdot \cos\chi \cdot \rho \sin\alpha \; d\rho \; d\eta$$

Torque Due to Radiation Pressure (cont)

- The radius vector is, in conical coordinates,

$$\vec{r} = \mathbb{R}(\alpha,\eta)^{-1}\begin{bmatrix} \rho\sin\alpha\cos\eta \\ \rho\sin\alpha\sin\eta \\ h+\dfrac{a}{\tan\alpha}-\rho\cos\alpha \end{bmatrix} = \begin{bmatrix} -h\cos\alpha - a\frac{\cos^2\alpha}{\sin\alpha}+\rho \\ 0 \\ h\sin\alpha + a\cos\alpha \end{bmatrix}$$

- Perform the first integral to get

$$\begin{bmatrix} K_\rho \\ K_\eta \\ K_\alpha \end{bmatrix} = P(b-a)\int_0^{2\pi}\begin{bmatrix} -B_2\pi_\eta \\ B_2\pi_\rho + B_1 Q\,\pi_\alpha\cos\chi - 2B_1 A_C\cos\chi \\ B_2(1-A_C)P_\eta\cos\chi \end{bmatrix}\cos\chi\,d\eta$$

where

$$B_1 \equiv \tfrac{1}{2}\tfrac{1}{\sin^2\alpha}[U(a+b)\cos\alpha - \tfrac{2}{3}(a^2+ab+b^2)]$$

$$B_2 \equiv \tfrac{1}{2}\tfrac{1}{\sin\alpha}QU(a+b)$$

$$Q \equiv 1-A_C \qquad U \equiv h\sin\alpha + a\cos\alpha$$

Torque Due to Radiation Pressure (cont)

- Transform back to the cartesian body frame:

$$\begin{bmatrix} K_x \\ K_y \\ K_z \end{bmatrix} = P(b-a)\int_0^{2\pi} \mathbb{R}(\alpha,\eta) \begin{bmatrix} -B_2\pi_\eta \\ B_2\pi_\rho + B_1 Q\,\pi_\alpha \cos\chi - 2B_1 A_C \cos\chi \\ B_2(1-A_C)P_\eta \cos\chi \end{bmatrix} \cos\chi\, d\eta$$

- Perform the integral to get, finally, the total torque acting on the conical surface:

$$\begin{bmatrix} K_x \\ K_y \\ K_z \end{bmatrix} = V \begin{bmatrix} \pi_X(\cos\psi\cos\theta\sin\phi + \sin\theta\cos\phi) \\ \quad + \pi_Y(\sin\theta\sin\phi - \cos\psi\cos\theta\cos\phi) - \pi_Z\cos\theta\sin\psi \\ \pi_X(\cos\theta\cos\phi - \cos\psi\sin\theta\sin\phi) \\ \quad + \pi_Y(\cos\theta\sin\phi + \cos\psi\sin\theta\cos\phi) + \pi_Z\sin\theta\sin\psi \\ 0 \end{bmatrix}$$

where

$$\begin{aligned} V &\equiv \pi P(b-a)(-\pi_X\sin\psi\sin\phi + \pi_Y\sin\psi\cos\phi - \pi_Z\cos\psi) \\ &\quad \cdot [B_1(3+A_C)\cos\alpha\,\sin\alpha - B_2(2\sin^2\alpha - \cos^2\alpha)] \end{aligned}$$

$$B_1 \equiv \tfrac{1}{2}\tfrac{1}{\sin^2\alpha}[U(a+b)\cos\alpha - \tfrac{2}{3}(a^2+ab+b^2)]$$

$$B_2 \equiv \tfrac{1}{2}\tfrac{1}{\sin\alpha}QU(a+b)$$

$$Q \equiv 1 - A_C \qquad U \equiv h\sin\alpha + a\cos\alpha$$

Torque Due to Radiation Pressure (cont)

- The contribution to the "top" surface is just a special case of the cone formulation. Hence, we find

$$\begin{bmatrix} K_x \\ K_y \\ K_z \end{bmatrix} = W \begin{bmatrix} \pi_X(\cos\psi\cos\theta\sin\phi + \sin\theta\cos\phi) \\ \quad + \pi_Y(\sin\theta\sin\phi - \cos\psi\cos\theta\cos\phi) - \pi_Z\cos\theta\sin\psi \\ \pi_X(\cos\theta\cos\phi - \cos\psi\sin\theta\sin\phi) \\ \quad + \pi_Y(\cos\theta\sin\phi + \cos\psi\sin\theta\cos\phi) + \pi_Z\sin\theta\sin\psi \\ 0 \end{bmatrix}$$

where

$$W \equiv \pi P a^2 h (1 - A_T)(-\pi_X \sin\psi \sin\varphi + \pi_Y \sin\psi \cos\varphi - \pi_Z \cos\psi)$$

The Equations of Motion

- Therefore, the equations of motion become

$$
\begin{aligned}
\frac{d\varphi}{dt} &= \Omega_\varphi \\
\frac{d\psi}{dt} &= \Omega_\psi \\
\frac{d\varphi}{dt} &= \Omega_\varphi \\
\sin\psi \frac{d}{dt}\Omega_\varphi &= [(1-\beta)\,\Omega_\theta - (1+\beta)\cos\psi\,\Omega_\varphi]\,\Omega_\psi + K_1(a,b,h,\alpha,A_C,A_T,\varphi,\psi) \\
\frac{d}{dt}\Omega_\psi &= \left[\beta\cos\psi\,\Omega_\varphi^2 - (1-\beta)\,\Omega_\theta\,\Omega_\varphi\right]\sin\psi + K_2(a,b,h,\alpha,A_C,A_T,\varphi,\psi) \\
\sin\psi \frac{d}{dt}\Omega_\theta &= [(1+\beta\cos^2\psi)\,\Omega_\varphi - (1-\beta)\cos\psi\,\Omega_\theta]\,\Omega_\psi + K_3(a,b,h,\alpha,A_C,A_T,\varphi,\psi)
\end{aligned}
$$

$$
\begin{aligned}
K_1(a,b,h,\alpha,A_C,A_T,\varphi,\psi) &\equiv G(a,b,h,\alpha,A_C,A_T)\cdot g_1(\varphi,\psi) \\
K_2(a,b,h,\alpha,A_C,A_T,\varphi,\psi) &\equiv G(a,b,h,\alpha,A_C,A_T)\cdot g_2(\varphi,\psi) \\
K_3(a,b,h,\alpha,A_C,A_T,\varphi,\psi) &\equiv G(a,b,h,\alpha,A_C,A_T)\cdot g_3(\varphi,\psi)
\end{aligned}
$$

$$
\begin{aligned}
g_0(\varphi,\psi) &= -\pi_X \sin\varphi\,\sin\psi + \pi_Y \sin\psi\,\cos\varphi - \pi_Z\cos\psi \\
g_1(\varphi,\psi) &= \quad g_0(\varphi,\psi)\cdot(\pi_X\cos\varphi + \pi_Y\sin\varphi) \\
g_2(\varphi,\psi) &= \quad g_0(\varphi,\psi)\cdot(\pi_X\cos\psi\,\sin\varphi - \pi_Y\cos\psi\,\cos\varphi - \pi_Z\sin\psi) \\
g_3(\varphi,\psi) &= -g_1(\varphi,\psi)\cos\psi
\end{aligned}
$$

$$
\begin{aligned}
G(a,b,h,\alpha,A_C,A_T) &= G_C(a,b,h,\alpha,A_C) + G_T(a,h,A_T) \\
G_C(a,b,h,\alpha,A_C) &= \frac{\pi P}{I_{xy}}(b-a)\Big[(1-A_C+2A_C\cos^2\alpha)(h\sin\alpha + a\cos\alpha)\frac{a+b}{\sin\alpha} \\
&\qquad - \tfrac{1}{3}(3+A_C)\cos\alpha\,\frac{a^2+ab+b^2}{\sin\alpha}\Big] \\
G_T(a,h,A_T) &= \frac{\pi P}{I_{xy}}(1-A_T)a^2h
\end{aligned}
$$

Basic Behavior

Fast Spin Approximation — Precession

- Assume a fast-spinning craft. Then $\Omega_\theta >> \Omega_\varphi, \Omega_\psi$ Differentiate, substitute, and perform a series expansion on Ω_ψ, Ω_φ, and *P* to get

$$\begin{aligned}
\sin\psi \frac{d^2}{dt^2}\Omega_\varphi &= -(1-\beta)^2 \Omega_\theta^2 \Omega_\varphi \sin\psi + (1-\beta)\Omega_\theta K_2(a,b,h,\alpha,A_C,A_T,\varphi,\psi) \\
\frac{d^2}{dt^2}\Omega_\psi &= -(1-\beta)^2 \Omega_\theta^2 \Omega_\psi - (1-\beta)\Omega_\theta K_1(a,b,h,\alpha,A_C,A_T,\varphi,\psi) \\
\sin\psi \frac{d^2}{dt^2}\Omega_\theta &= [(1-\beta)^2 \Omega_\theta^2 \Omega_\varphi \sin\psi - (1-\beta)\Omega_\theta K_2(a,b,h,\alpha,A_C,A_T,\varphi,\psi)]\cos\psi
\end{aligned}$$

- From the first or third equations, we have

$$\Omega_\varphi \approx \frac{K_2(a,b,h,\alpha,A_C,A_T,\varphi,\psi)}{(1-\beta)\Omega_\theta \sin\psi}$$

- For a flat disk of uniform albedo A, this reduces to

$$\Omega_\varphi \approx \frac{(1-A)\pi b^2 h}{(1-\beta) I_{xy} \Omega_\theta} P \cos\psi$$

  which agrees with your average #10 business envelope.

  - Note dependencies:
    - area of shield
    - magnitude of pressure
    - cosine of sun angle
    - inverse of spin rate

Fast Spin Approximation — Nutation

- From the second equation we have simple harmonic motion for Ω_ψ:

$$\Omega_\psi \approx A\cos[(1-\beta)\,\Omega_\theta\, t] + B\sin[(1-\beta)\,\Omega_\theta\, t] - \frac{K_1(a,b,h,\alpha,A_C,A_T,\varphi,\psi)}{(1-\beta)\,\Omega_\theta}$$

  This is nutation.

- The third term is constant for $\pi_X, \pi_Y \to 0,\ \pi_Z \to .1$ This implies a small, monotonic drift in the inclination angle ψ. However, it is actually the first term in the expansion of a large-period oscillation.

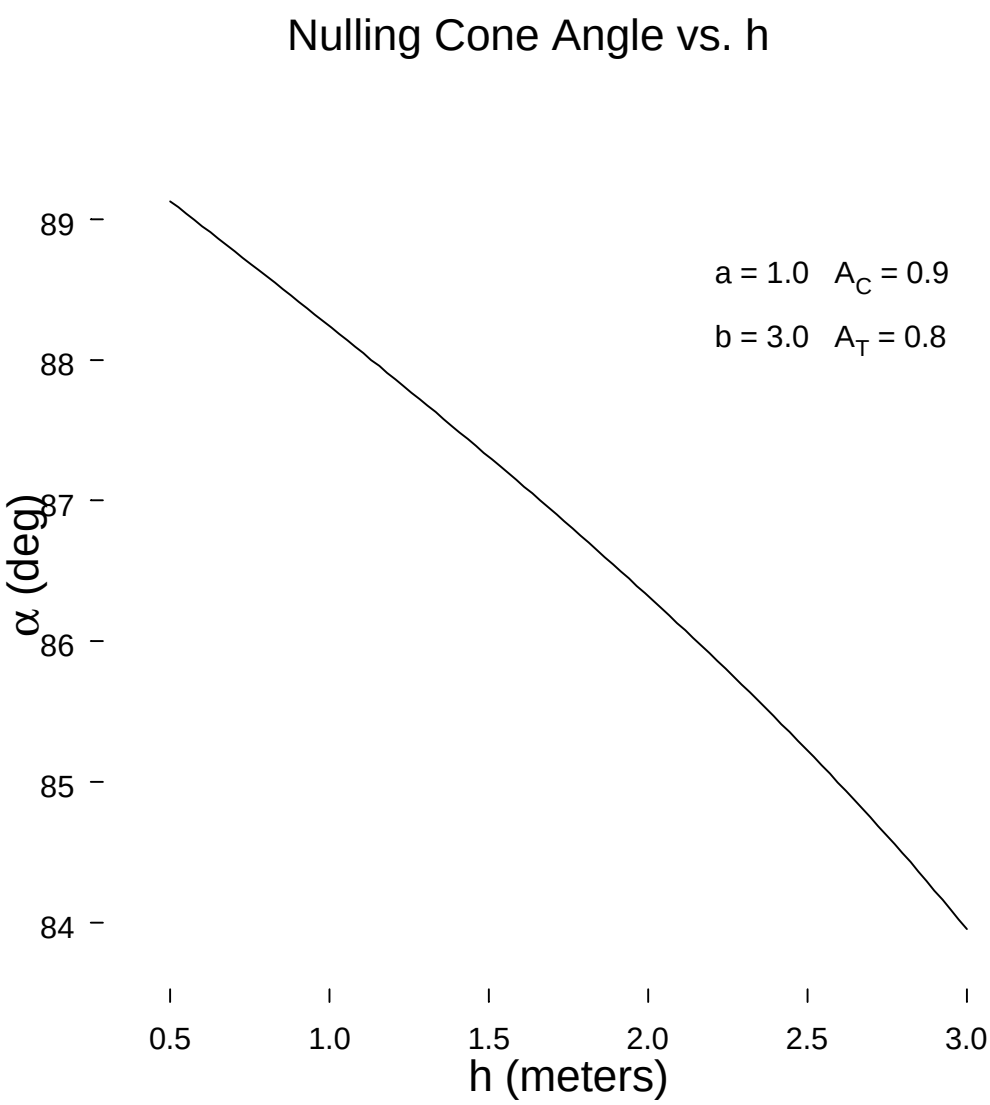
Cone Angle for Precession Nulling

- Precession null:

$$G(a, b, h, \alpha, A_C, A_T) = 0$$

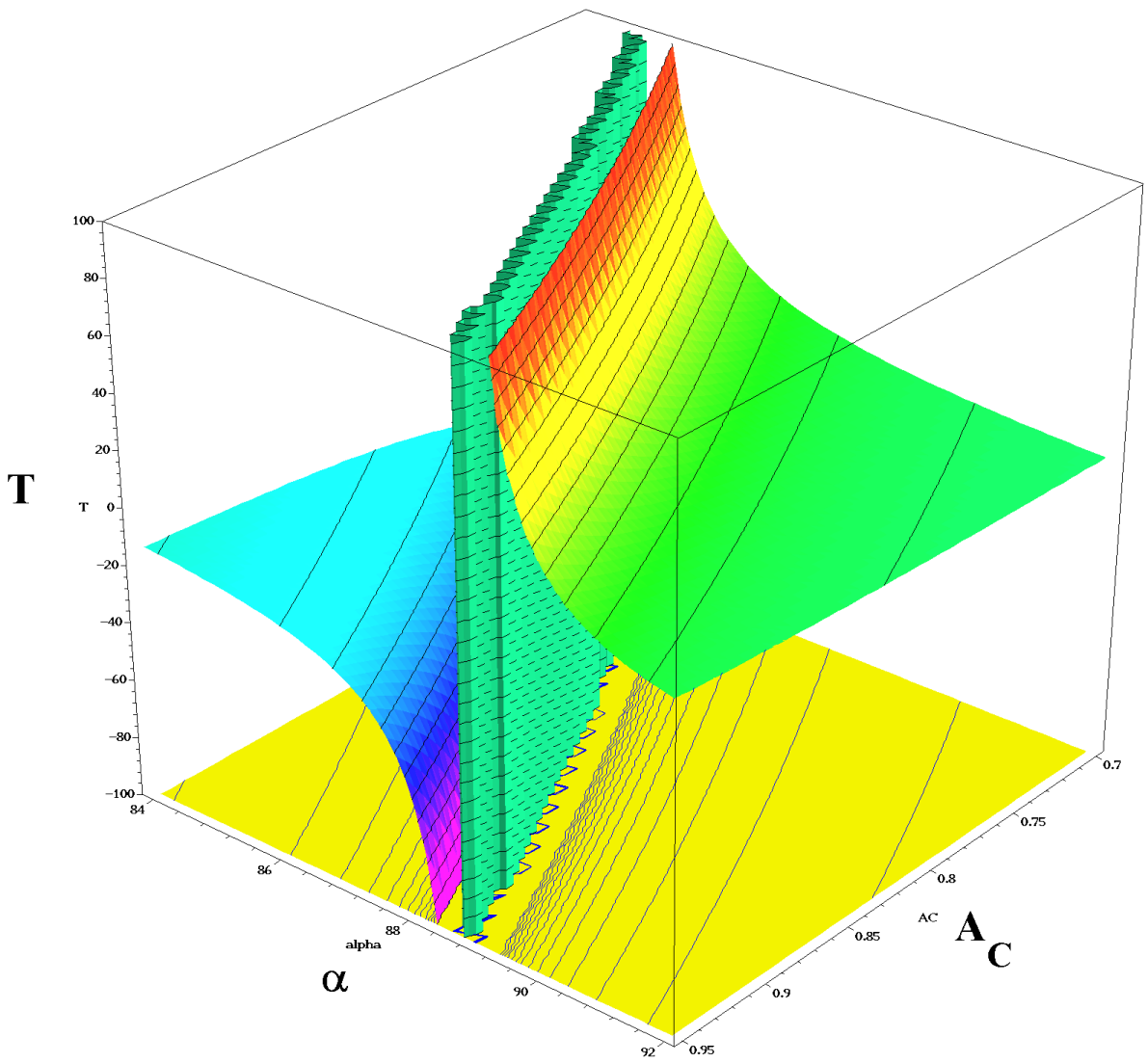
- This is equivalent to

$$(b-a)[(1-A_C+2A_C\cos^2\alpha)(h\sin\alpha+a\cos\alpha)(a+b) - \tfrac{1}{3}(3+A_C)(a^2+ab+b^2)\cos\alpha\,] + (1-A_T)a^2h\sin\alpha = 0$$

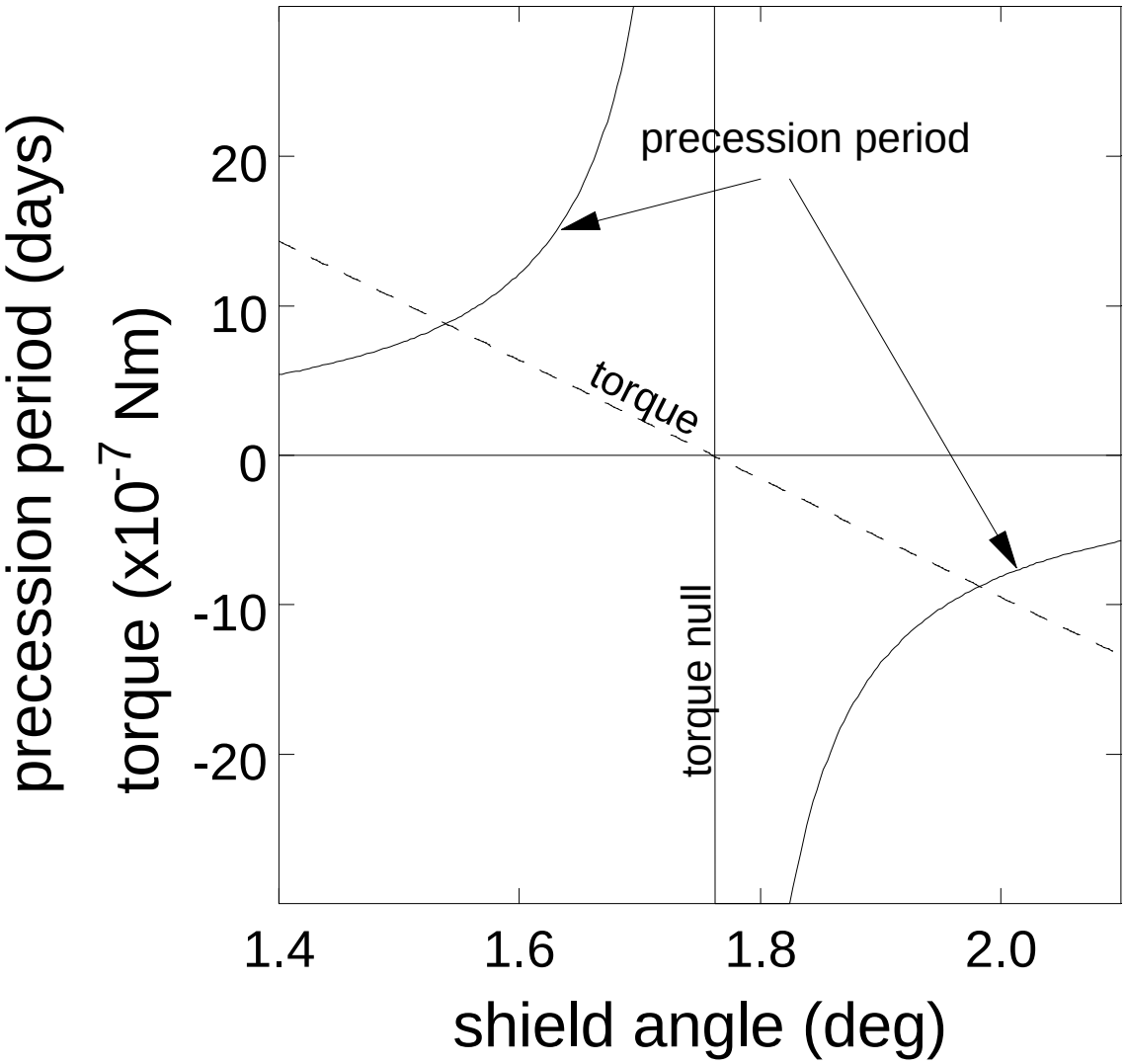


Precession Period vs. (a,A_C)

- Shield angle will need to be adjustable in flight, at least at beginning of mission.



Precession Period as Function of Shield Angle

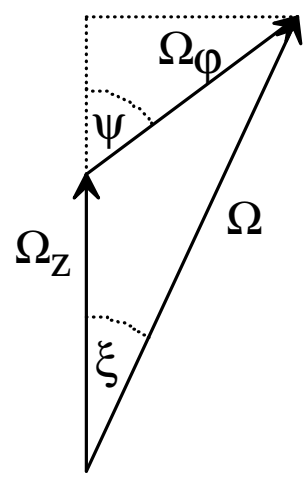


Basic Behavior

- Constant pressure field
  - Smooth precession $\Omega_\varphi \propto \dfrac{P\cos\psi}{(1-\beta)\Omega_\theta}$
  - Constant sun angle ψ
  - Thump it, and it rings (nutation)
  - Angular velocity along spin axis is conserved

$$\frac{d}{dt}\left(\Omega_\theta + \Omega_\varphi \cos\psi\right) = \frac{d}{dt}\Omega_z = 0$$

  - Angular velocity vector in the rotating spacecraft frame executes circular motion around the symmetry axis
    - Frequency $\Omega_z \pm \Omega_\varphi \cos\psi$
    - Radius $\tan\xi = \dfrac{\Omega_\varphi \sin\psi}{\Omega_z + \Omega_\varphi \cos\psi}$

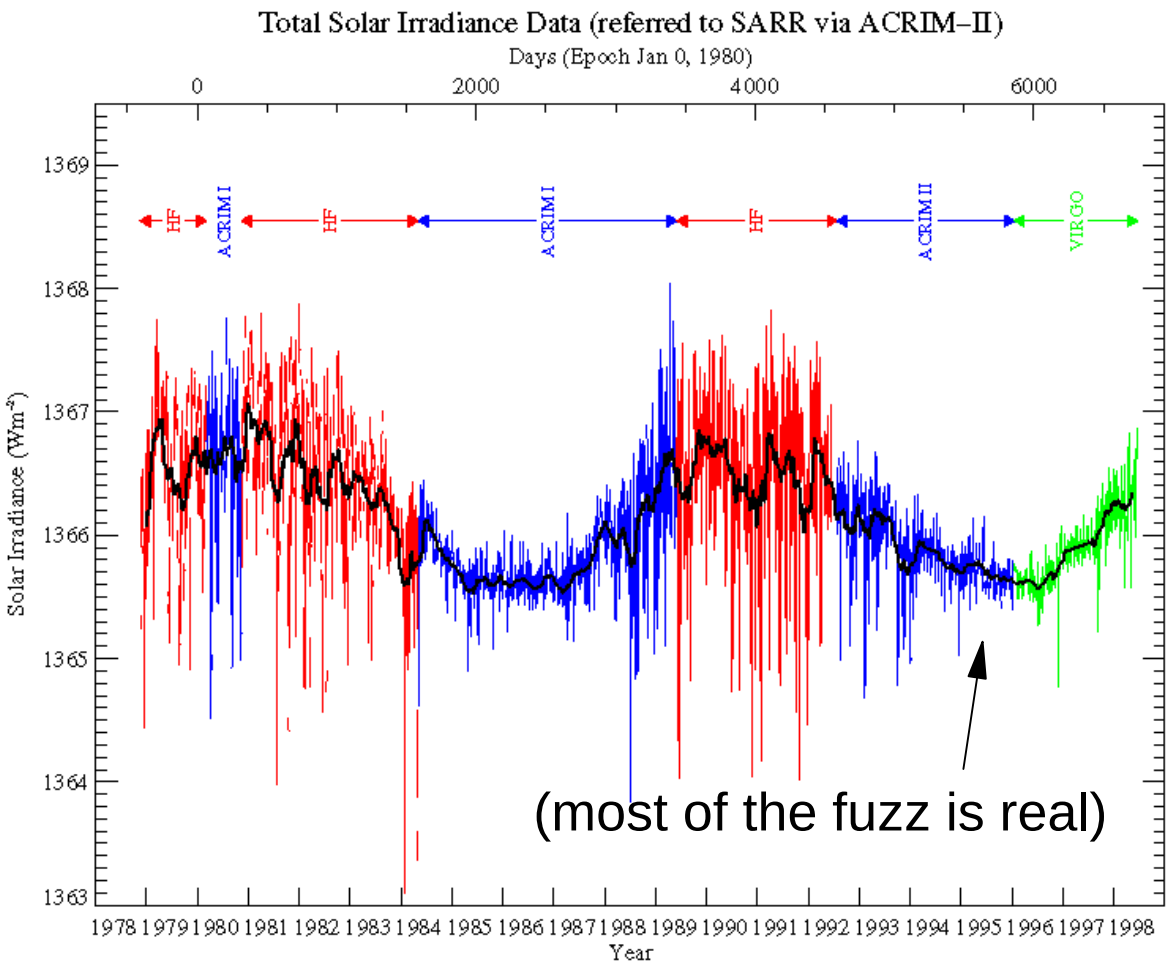


- Time-variable pressure field
  - Sun angle variations
    - variable-amplitude nutation
  - Precession variations
  - These variations result in "Pole wandering"
  - Spin (Ω_Z) unaffected, since for solar radiation there is no net torque component along the body z axis

Solar Irradiance Variations

Solar Irradiance Variations

- Irradiance variations 1978-1998



from: C. Fröhlich & J. Lean, 1998, Geoph.Res.Let., 25, pp.4377–4380, and the VIRGO Team (Nov 29, 1998)

Solar Irradiance Variations (continued)

- Stochastic at the 0.1 percent level
- Cannot be measured accurately enough or with the required time resolution (a few minutes) from the ground
- Existing satellite data (WIND, ACRIM I & II, etc.): not enough time resolution
- SoHO/VIRGO to the rescue
  - Best accuracy of any instrument to date
  - 1-minute time resolution
- Problem: VIRGO team in intense competition against other SoHO instrument teams
  - Holy grail: solar g-mode oscillations
  - To get data, had to state our case all the way up the chain from American team members to the European VIRGO PI

Solar Irradiance Variations (continued)

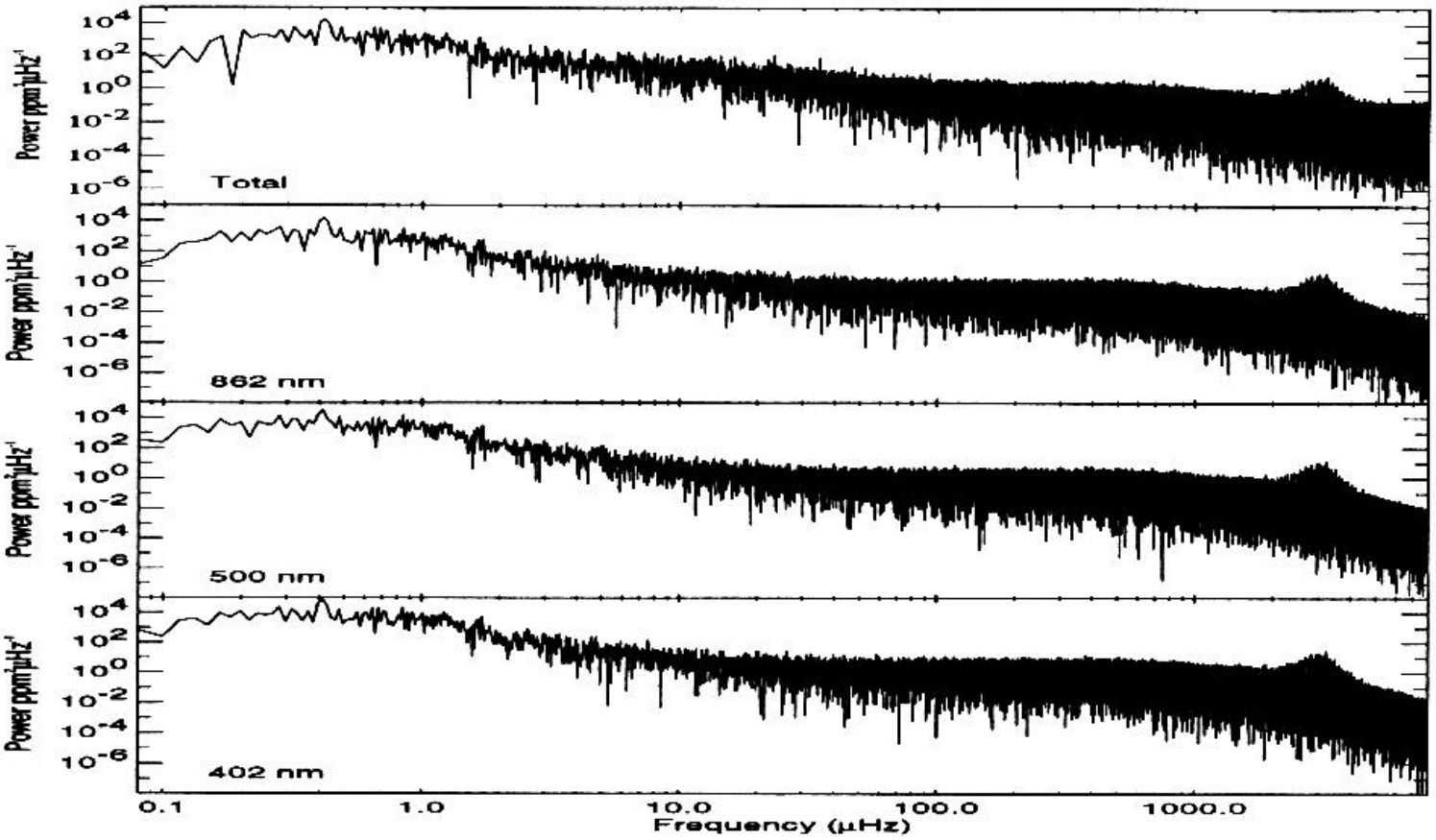
- Resolution: the VIRGO team is kind, trusting, and reasonable
  - **We now have a full year of VIRGO 1-minute irradiance data**

Solar Irradiance Variations (continued)

- Characteristic irradiance variations
  - In general, power below 1 μHz is due to active regions.
  - Most of the signal is at very low frequencies, around 0.4-0.5 μHz (~23-30 days). This corresponds to the solar rotation synodic period of ~27 days and represents the variations due to sunspot and plague regions rotating in and out of view.
  - There is a significant surge of signal in the range 2000-3000 μHz (~5 minutes). This signal is from irradiance variations due to the 5-minute solar p-mode oscillations.
  - The power in the region 10-100 μHz is due to supergranulation
  - The power in the region 80-1000 μHz is due to mesogranulation.
  - The power in the region 800-3000 μHz is due to granulation.
  - See J. Pap et al., 1999, *Adv. Space Res.*, in press.
  - High-order p-mode oscillations: coherence times of one to a few days (F. Varadi, 1999, private comm.)

Solar Irradiance Variations (continued)

- SoHO/VIRGO power spectrum

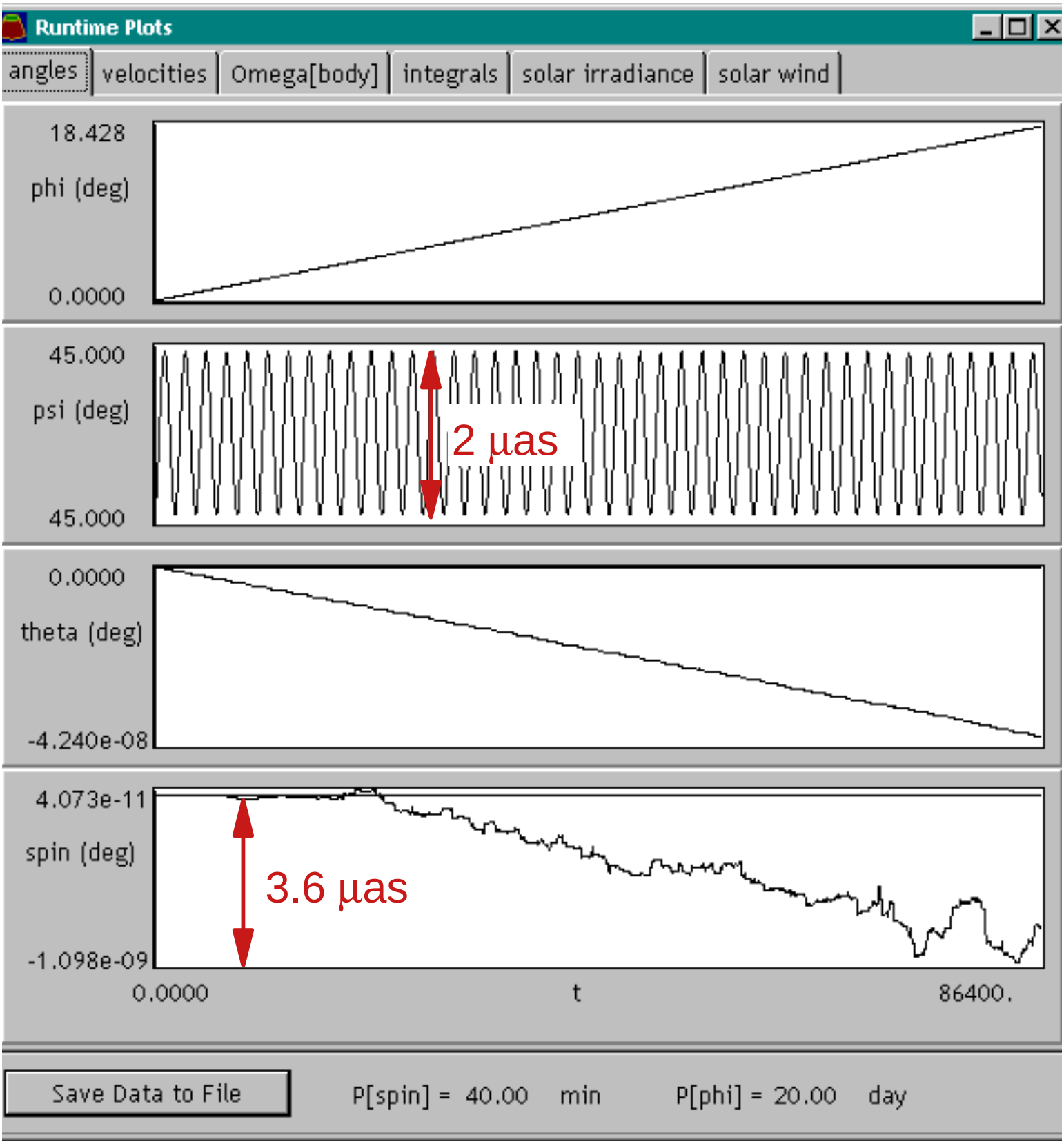


frequency (μHz)

Solar Irradiance Fluctuation Effects on Spacecraft Attitude

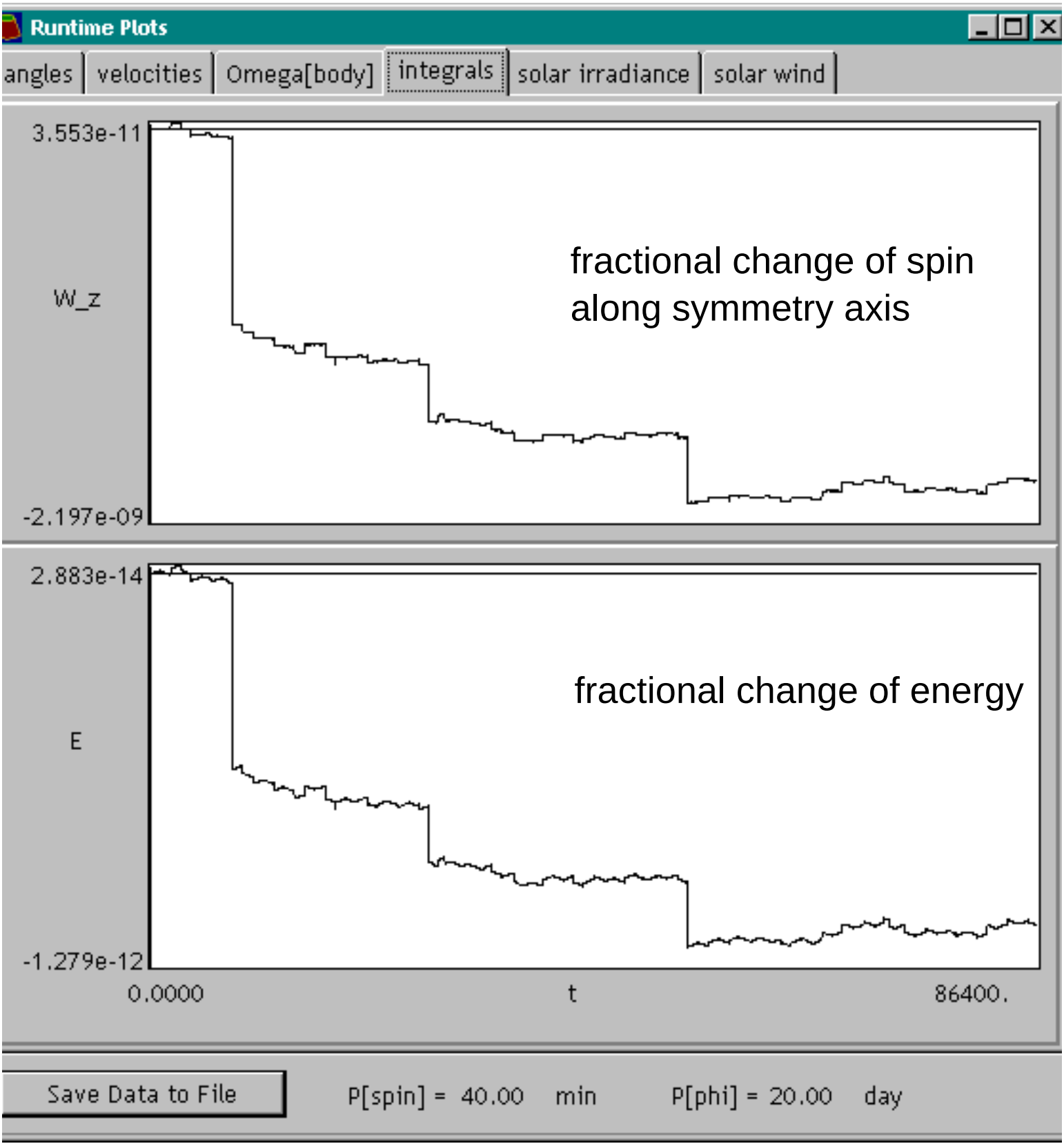
Solar irradiance fluctuation effects on spacecraft attitude

- Steady irradiance reference case



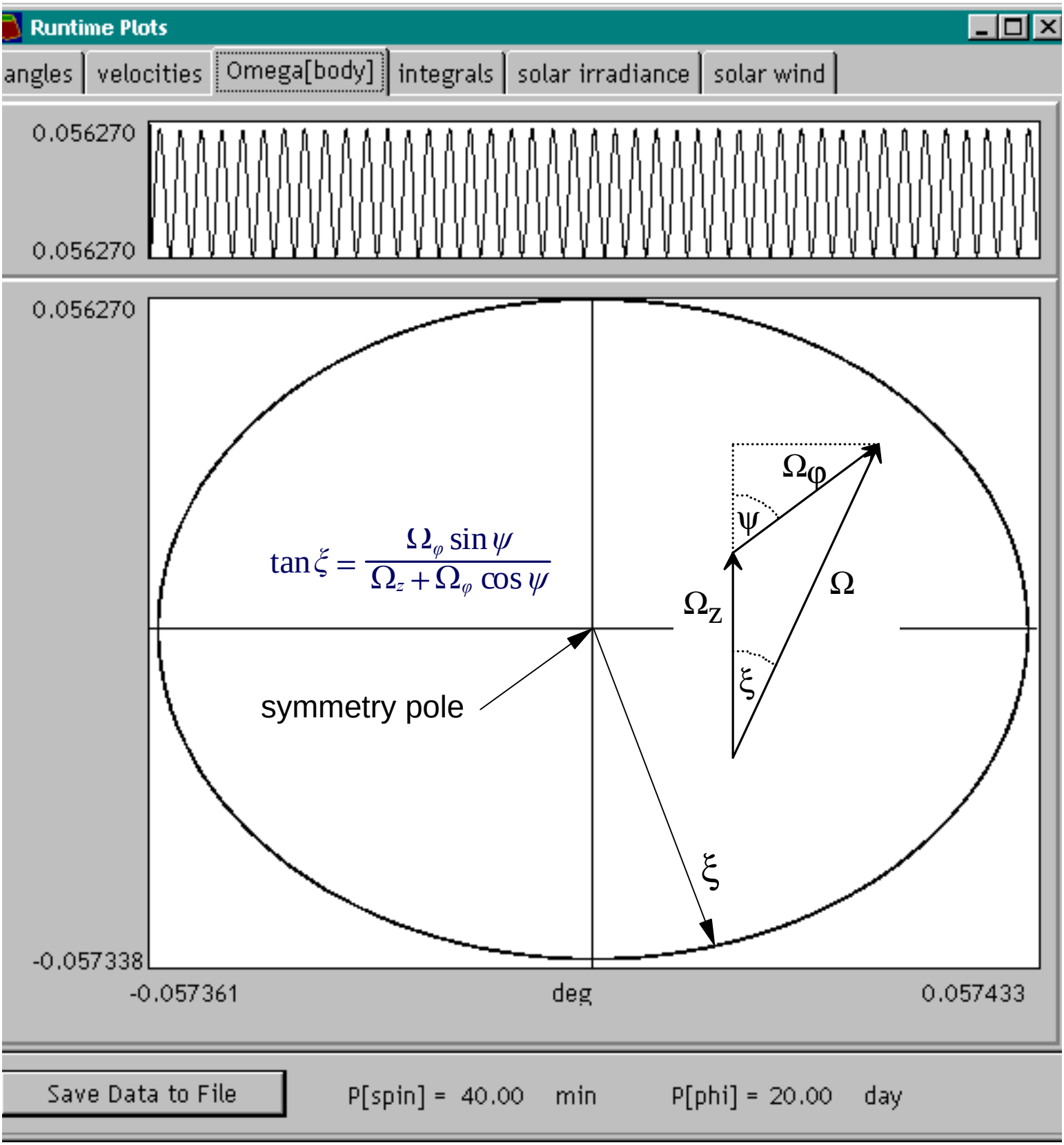
Solar irradiance fluctuation effects on spacecraft attitude (continued)

- Reference case: check on conserved quantities



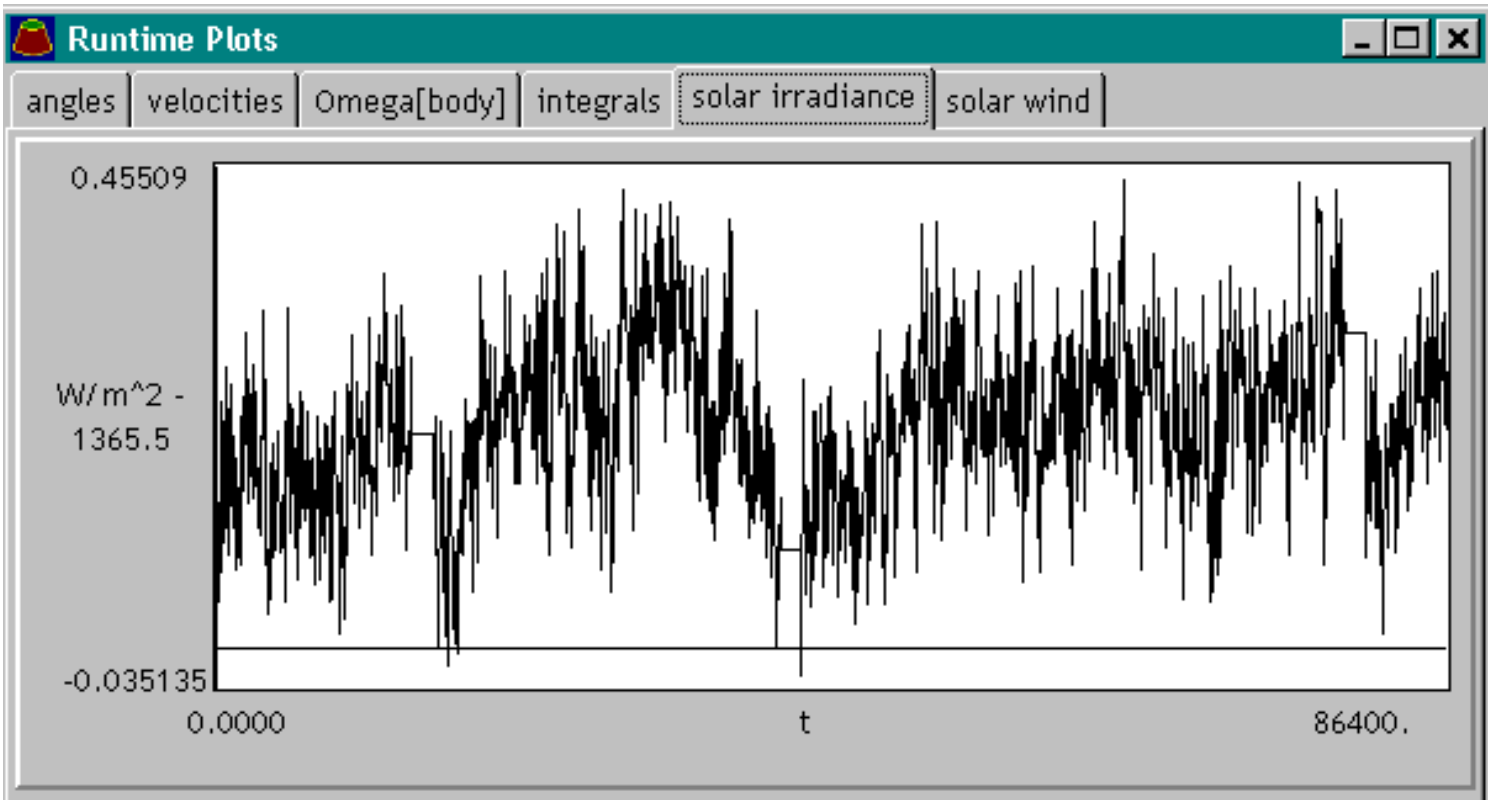
Solar irradiance fluctuation effects on spacecraft attitude (continued)

- Reference case: body-frame angular velocity



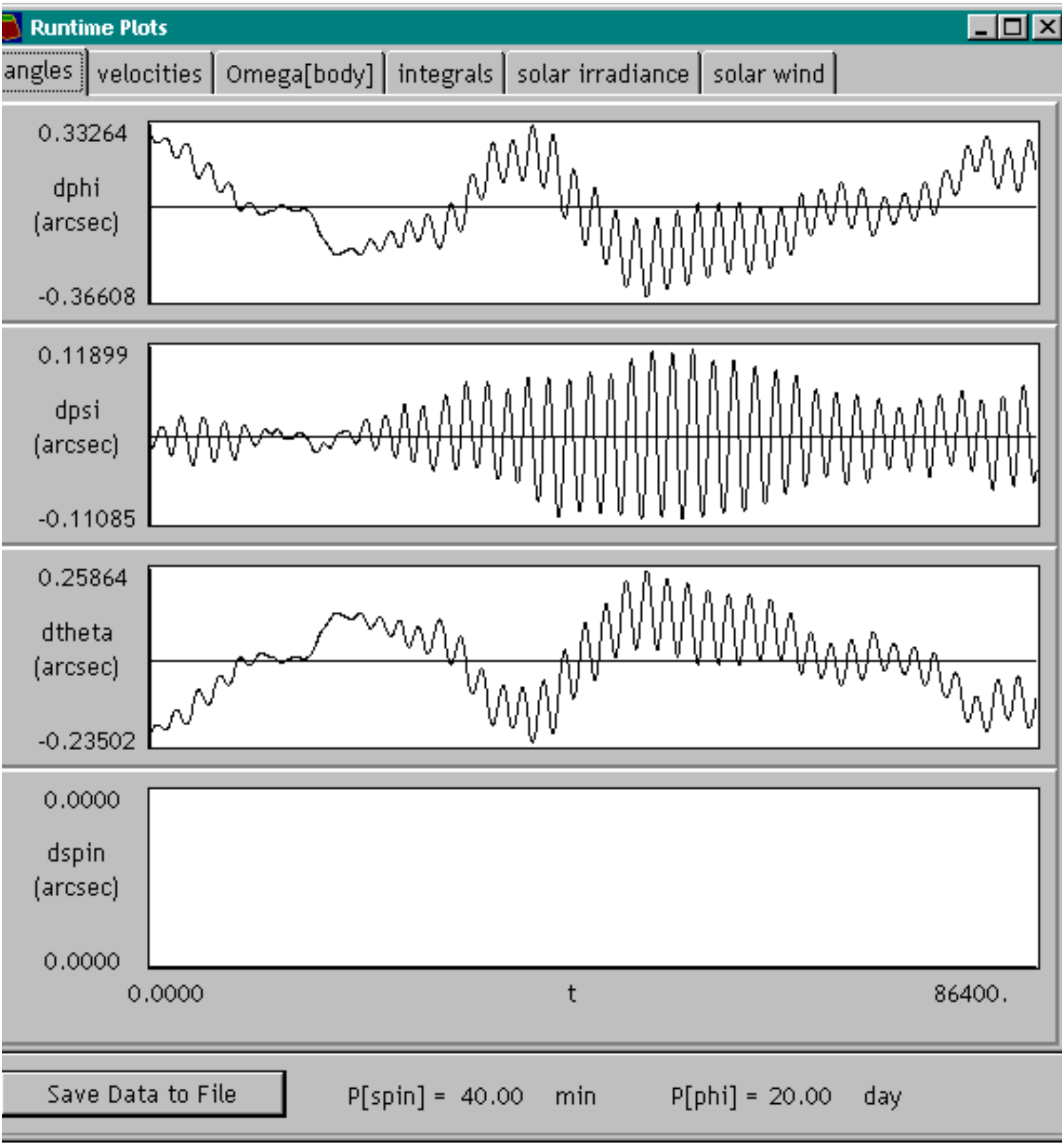
Solar irradiance fluctuation effects on spacecraft attitude (continued)

- Irradiance fluctuations



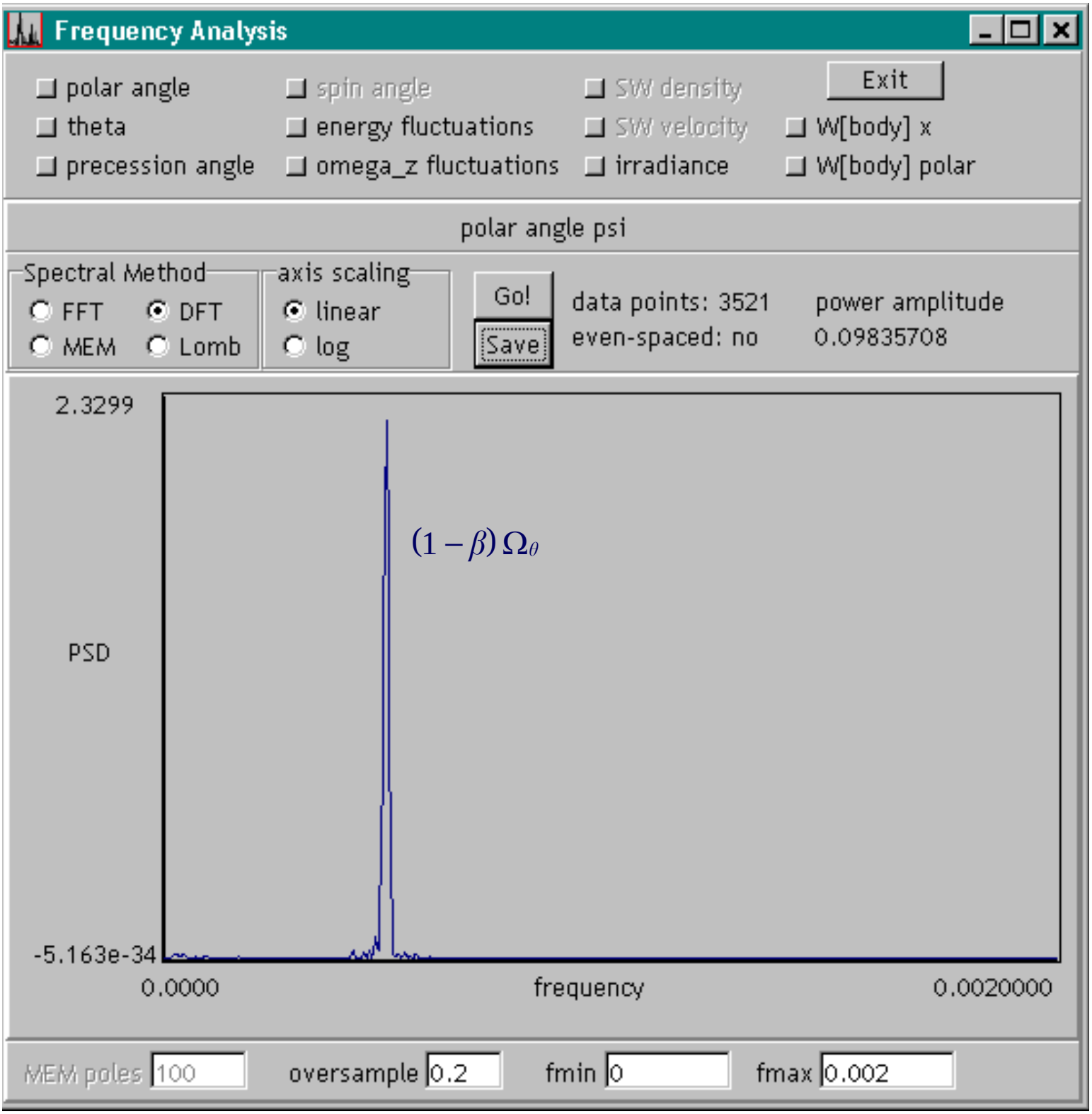
Solar irradiance fluctuation effects on spacecraft attitude (continued)

- Orientation differences from irradiance fluctuations



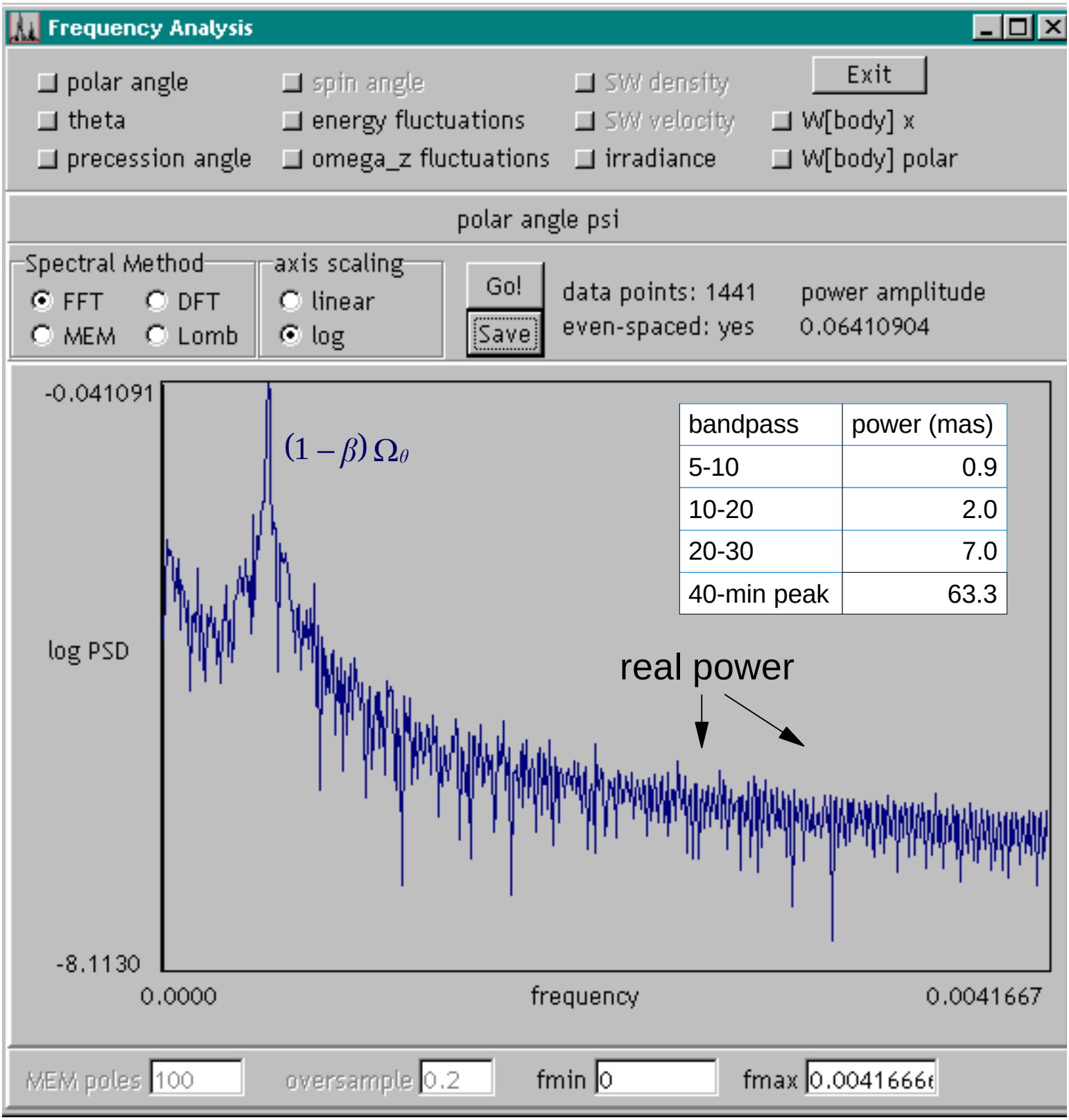
Solar irradiance fluctuation effects on spacecraft attitude (continued)

- Power spectral density of sun angle ψ



Solar irradiance fluctuation effects on spacecraft attitude (continued)

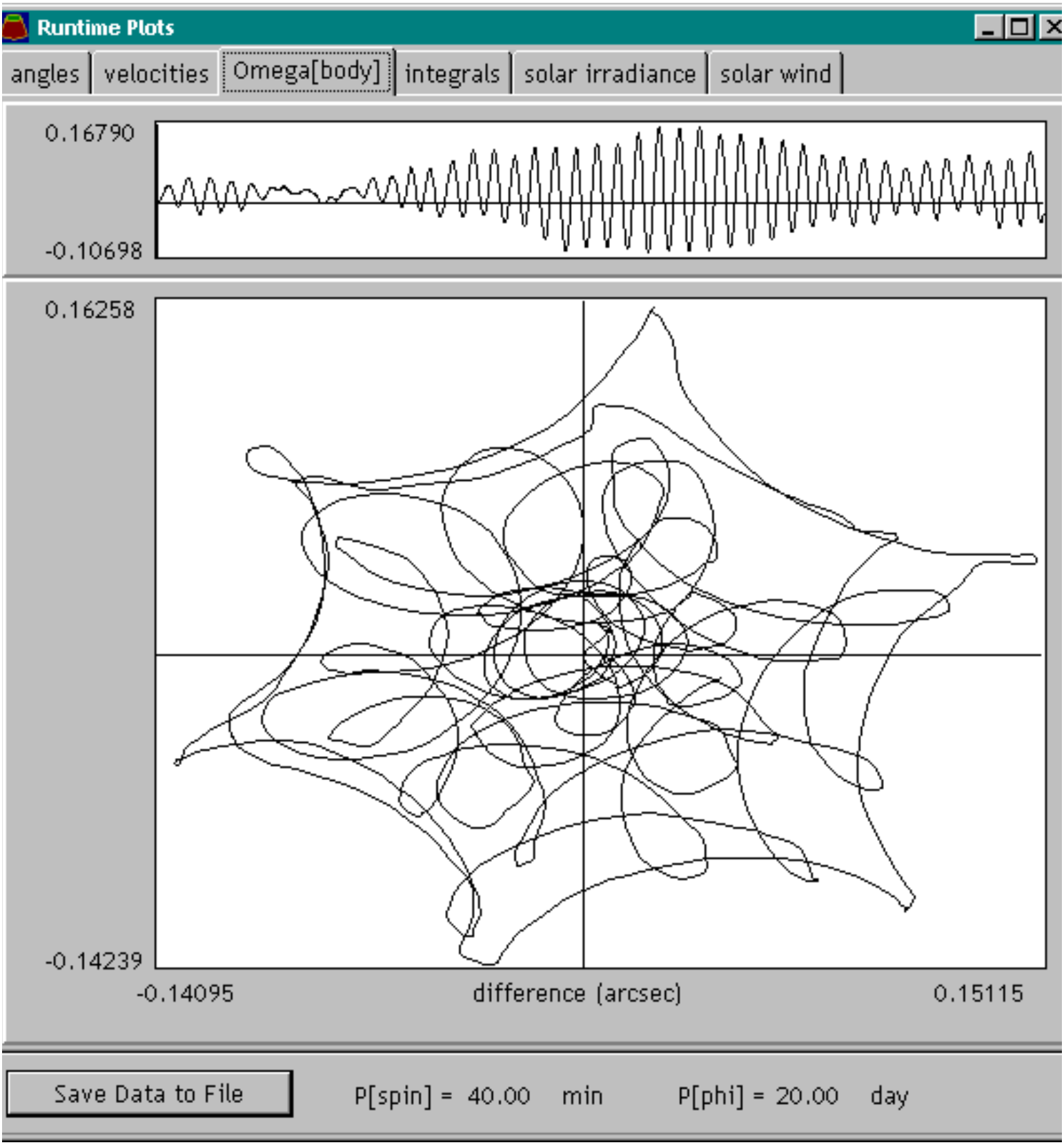
- Power spectral density of sun angle ψ (log scale)



bandpass	power (mas)
5-10	0.9
10-20	2.0
20-30	7.0
40-min peak	63.3

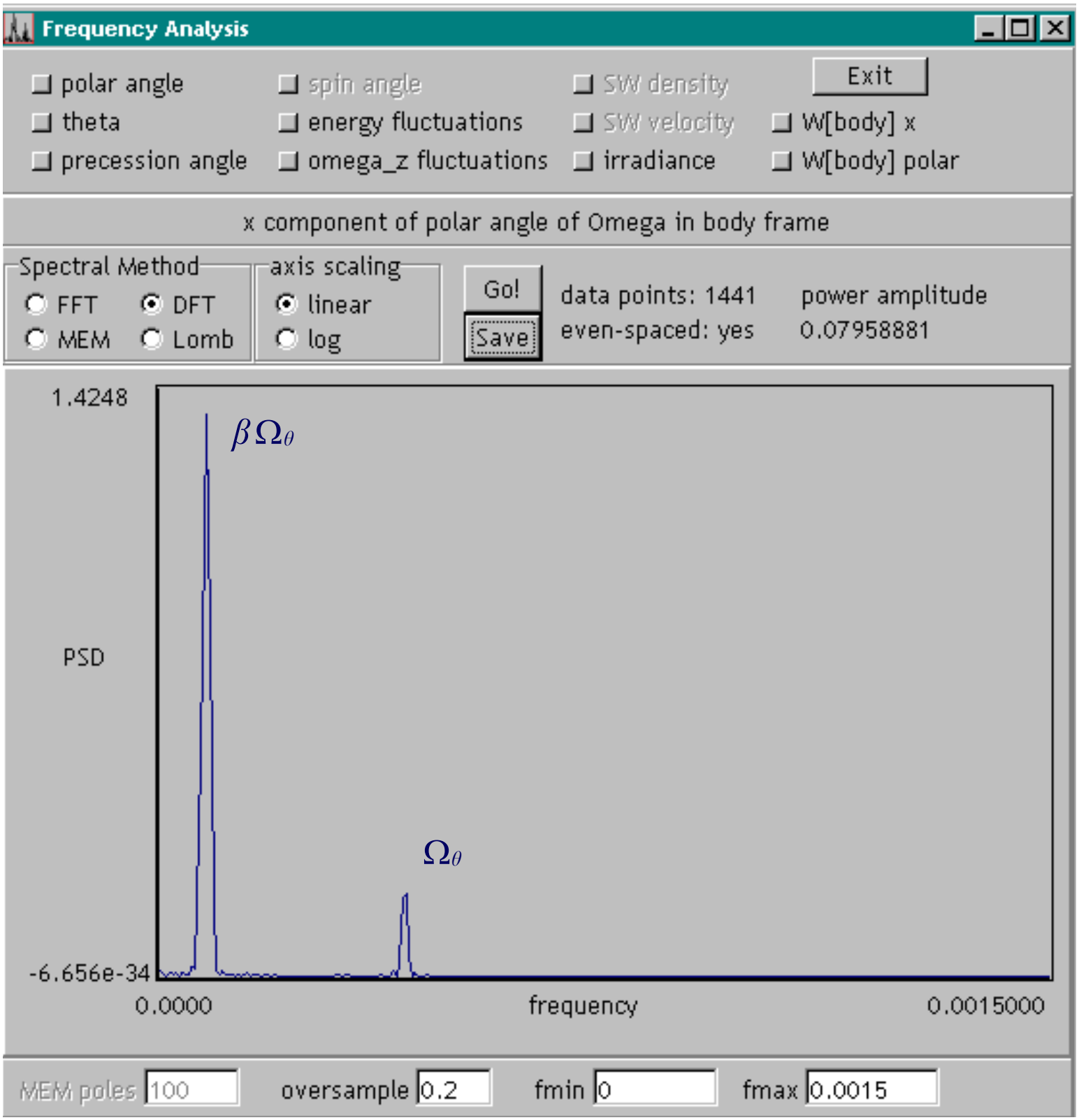
Solar irradiance fluctuation effects on spacecraft attitude (continued)

- body-frame angular velocity vector difference



Solar irradiance fluctuation effects on spacecraft attitude (continued)

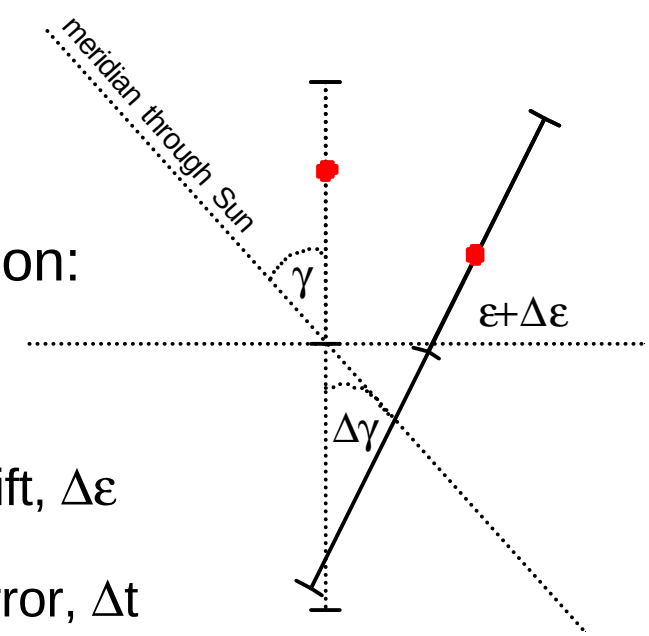
- body-frame angular velocity difference x-component PSD



Simulated Observations

Simulated Observations

- Installed arbitrary number of viewports (use 2), separated by a uniform angle (81.5 degrees)
- "Detect" a star when it crosses a fiducial line segment on the focal plane
  - Project line segment onto sky
- Observables:
  - time of detection
  - cross-scan location ε
- Effects of orientation fluctuation:
  - rotation of fiducial line, Δγ
  - cross-scan displacement shift, Δε
  - these give rise to a timing error, Δt



- Relation to integration variables (Euler angles θ,ψ,φ) is a simple problem in spherical trig.:

$$
\begin{aligned}
\tan\gamma &= \tan\psi\cos\theta \\
\cos\varepsilon\cos\kappa &= \sin\psi\sin\beta\sin\theta + (\sin(\lambda-\varphi)\cos\psi\sin\theta + \cos(\lambda-\varphi)\cos\theta)\cos\beta \\
\cos\varepsilon\sin\kappa &= \sin\psi\sin\beta\cos\theta + (\sin(\lambda-\varphi)\cos\psi\cos\theta - \cos(\lambda-\varphi)\sin\theta)\cos\beta \\
\sin\varepsilon &= \cos\psi\sin\beta - \sin(\lambda-\varphi)\sin\psi\cos\beta
\end{aligned}
$$

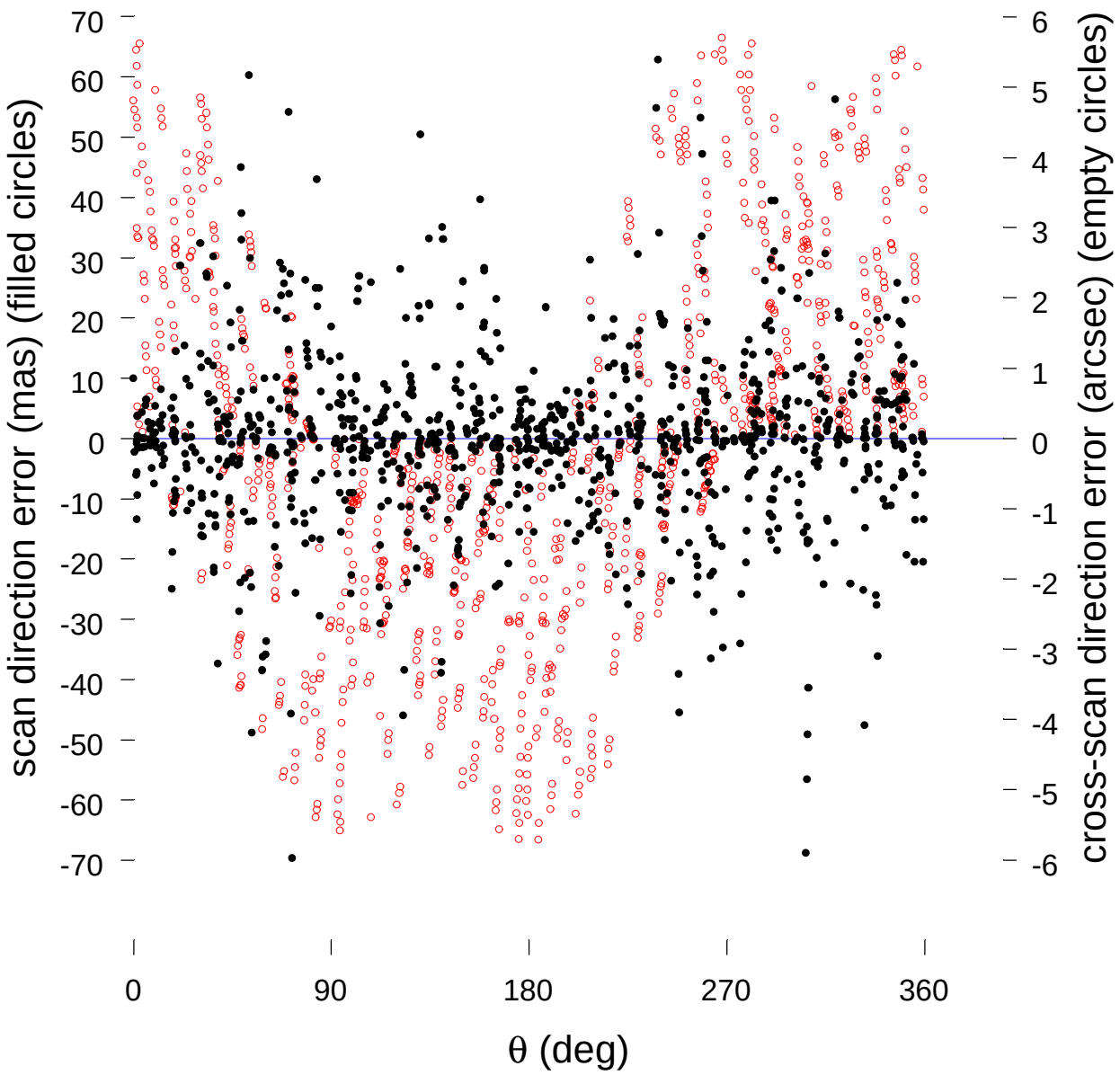
where k is azimuthal position of viewport

Simulated Observations (continued)

- Hence, we may "cheat" and take a look at $\Delta\varepsilon$ and Δt due to irradiance fluctuations
  - fluctuations events minus constant events

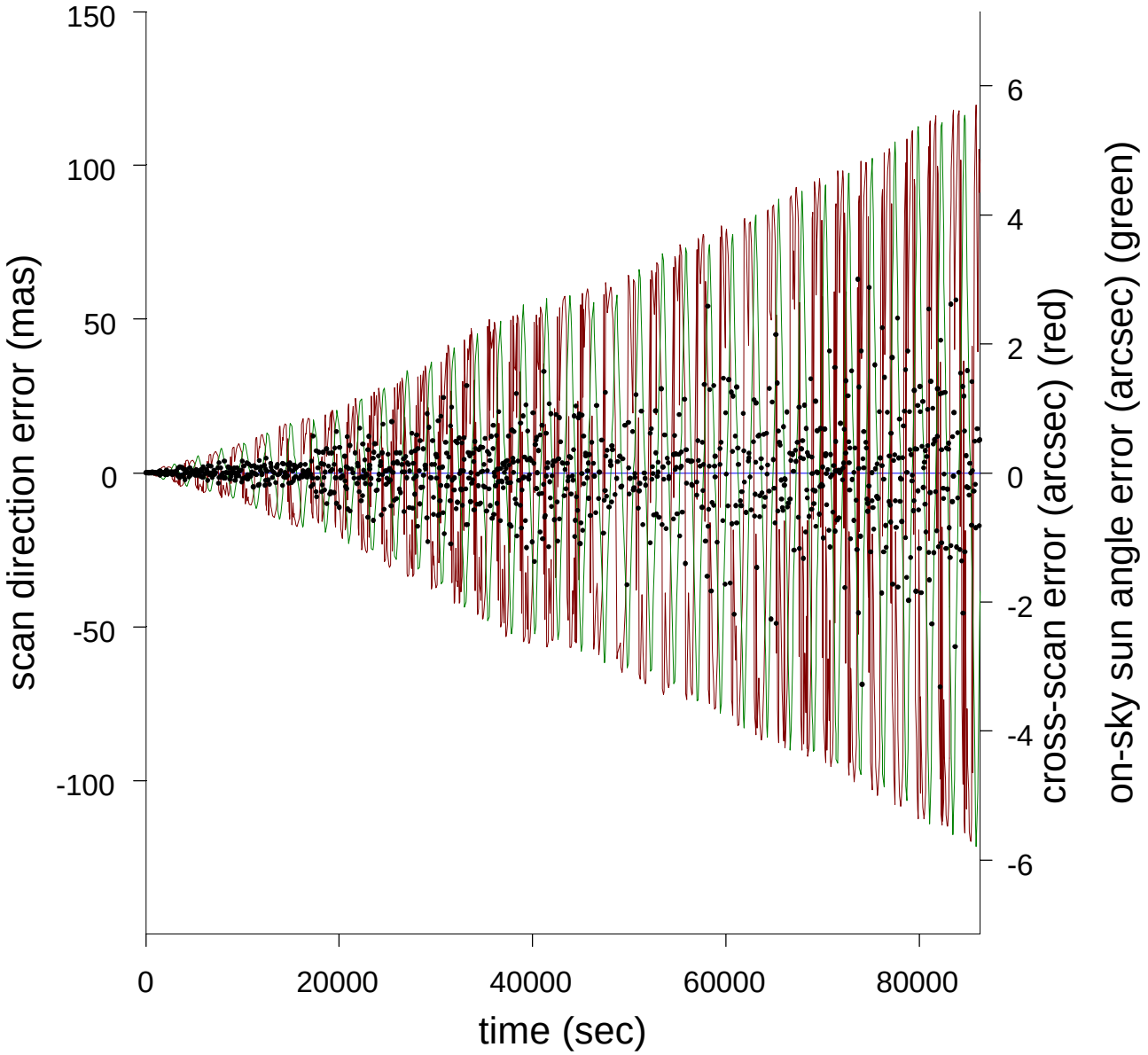
Simulated Observations (continued)

- Errors as a function of spin angle



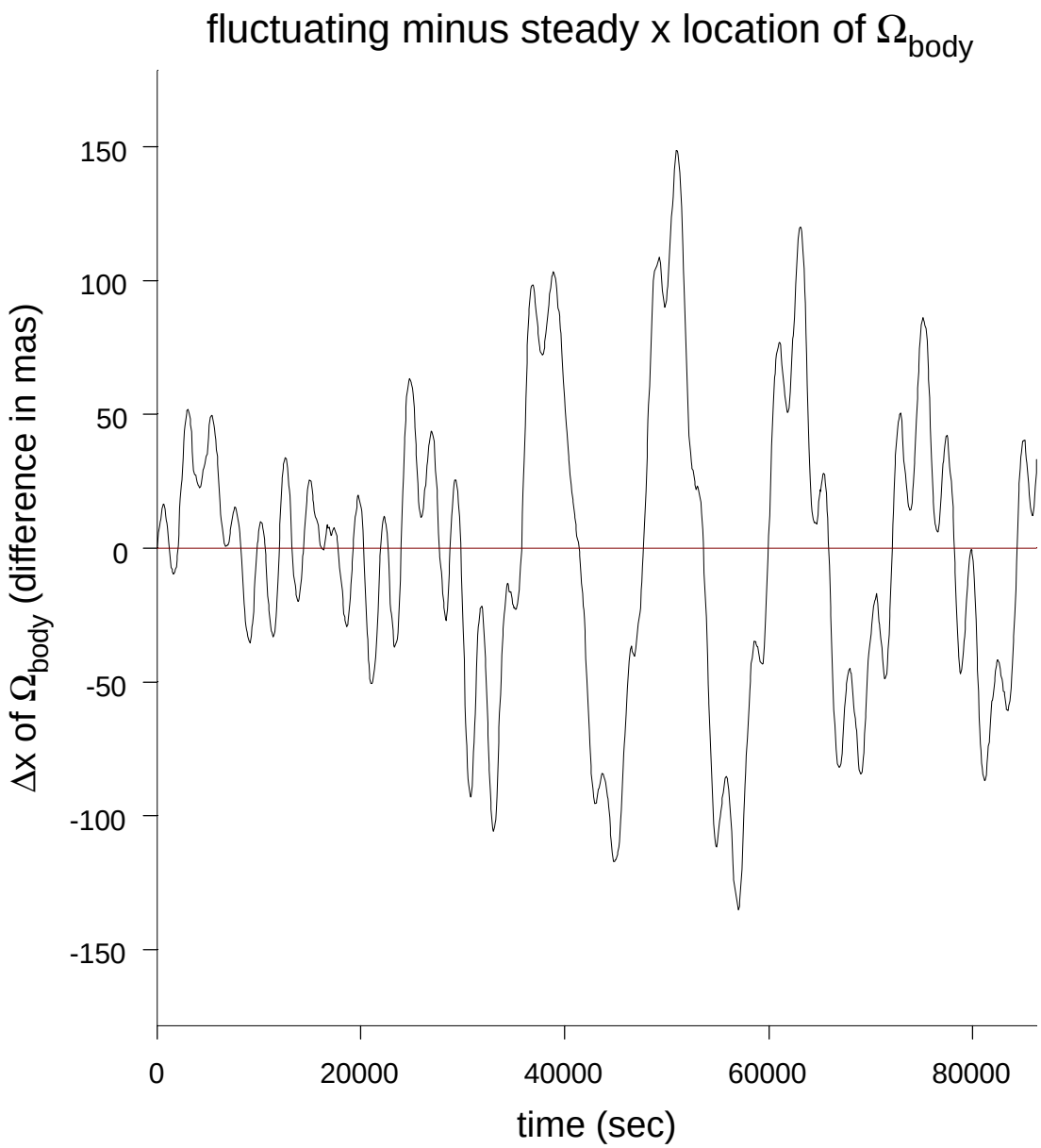
Simulated Observations (continued)

- Errors as a function of time



Simulated Observations (continued)

- Error in one component of angular velocity vector position in rotating body frame
  - variation is smooth
  - two dynamical frequencies
  - time-variable modulating envelope



Simulated Observations (continued)

- Questions
  - Why the quasi-linear growth with time?
  - Why the θ-dependence of the cross-scan errors?
  - [to be continued...]