Affects of Changing the Sun angle

Sean Urban

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1 Introduction

The question has recently been asked what affect does changing the angle between FAME's axis of rotation and the Sun, referred to in this document as the sun angle, ξ. The reasoning behind wanting this change is that a smaller sun angle than the 45° originally planned may allow the sun shield/solar array release and deployment system (SSRDS) to be simpler and, therefore, less risky. If we can tolerate a sun angle of $\approx 38^\circ$, then the SSRDS can use a one-hinged mechanism and still fit within the planned launch faring. This paper quantifies how the change in sun angle will affect astrometric performance.

2 The analysis

This question is specifically addressed in Høyer et al. (1981). The paper itself deals with the how varying certain parameters affects average astrometric mean errors in ecliptic coordinates and parallax. The authors run through some numerical experiments designed to test analytical formulae for the error propagation. These formulae are developed in the appendix found with in the same paper.

In my work, I have chosen not to use the numerical simulations, and instead utilize the equations found in the appendix. In broad terms, the simulations and the equations agree, but not precisely. The simulations do not have exactly what we need; sun angle is varied between 20° and 40° but the simulations beyond a $\xi = 30^\circ$ are not carried out long enough to derive parallaxes. The formulae in the appendix, however, allows the choice of many quantities in whatever increments you choose. The results in this memo are taken directly from the analysis in the appendix.

The authors state two assumptions in developing their formulae: 1) that strict observation equations must be simplified to where only terms which are essential to the *principle* of observation are retained; and 2) that sensible estimates can be obtained if certain quantities are more easily replace by easily computable averages (such as using an average number of observations per star than the actual number of observations for a given star).

Equation A.25 is

$$< \sigma_*^2 >= \frac{\varepsilon_\eta^2}{TC_*(\xi)}(1-Q_f)(1-Q_s)\times\left[1+\frac{n^{\frac{1}{3}}}{2(m-1)}\right]\times\frac{A\phi\frac{\varepsilon_\zeta}{\varepsilon_\eta}/\sqrt{m}}{\arctan(A\phi\frac{\varepsilon_\zeta}{\varepsilon_\eta}/\sqrt{m})} \quad (1)$$

It gives the variance of three astrometric parameters, β, λ, and Π, where β and λ are the equatorial latitude and longitude, and Π is parallax. The variance of each parameter (I use $*$ instead of the three Greek letters) is a function of:

- accuracy of observation in along scan direction, ε_η;
- accuracy of observation in cross-scan direction, ε_ζ;
- length of mission, T;
- weighting factor as function of sun angle, $C(\xi)$;
- overlap between frames, Q_f;
- fractional overlap between successive scans, Q_s;
- average number of stars per rotation, n;
- average number of stars in a frame, m;
- geometric factor as function of basic angle, $A(\gamma)$;
- and cross-scan FOV, Φ.

Assuming all other quantities remain unchanged, then the affect of changing the sun angle from ξ_1 to ξ_2 on the errors is

$$\frac{\sigma_{*_1}}{\sigma_{*_2}} = \sqrt{\frac{C_*(\xi_2)}{C_*(\xi_1)}} \tag{2}$$

where * is either β, λ, or Π. The weighting factor, C_*, is defined in and below equation A.24. Values for several different sun angles are found in Table A1 – reproduced below – and are plotted on the accompanying graphs.

ξ	$C_\beta(\xi)$	$C_\lambda(\xi)$	$C_\Pi(\xi)$
0	30.30	0.00	0.00
10	26.80	3.25	0.45
20	23.20	6.39	1.73
30	19.70	9.16	3.61
40	16.50	11.30	5.74
50	13.60	12.90	7.78
60	11.30	13.70	9.38
70	9.38	13.90	10.30
80	8.00	13.40	10.40
90	7.12	12.30	9.70

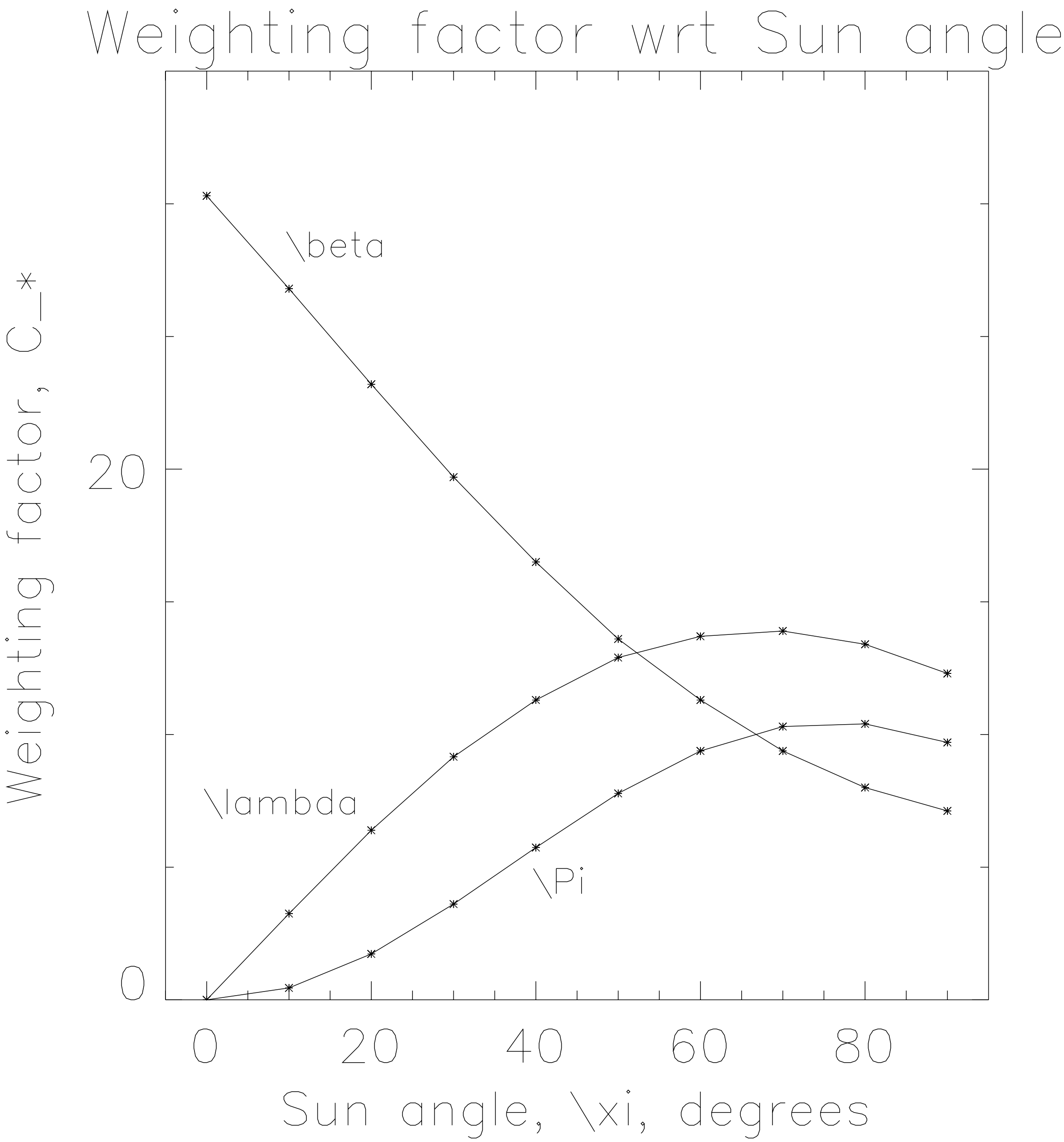
The effect of changing the sun angle from 45° to a smaller angle is to increase the accuracy in ecliptic latitude, but lower that accuracy in both ecliptic longitude and parallax. *Assuming* we can achieve a standard deviation of parallax,

σ_Π, of 50 μas, then the following table shows how good the errors in the other coordinates will be, according to the Høyer formulation. Additionally, it shows what changing the value of ξ does to all of these errors.

ξ	σ_β	σ_λ	σ_Π
45°	34	38	50
40°	32	39	55
38°	32	40	59
36°	31	41	60
35°	31	42	61

3 Reference

Høyer,P., Poder,K., Lindegren,L. & Høg,E. 1981, Derivation of Positions and Parallaxes from Simulated Observations with a Scanning Astrometry Satellite, A&A 110 228


Weighting factor wrt Sun angle
Weighting factor, C_*
Sun angle, \xi, degrees
0
20
0
20
40
60
80
\beta
\lambda
\Pi