2–dim fit precision and attitude determination

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Summary: no problems. A single 2–dim observation of a bright ($\approx 9^m$ to 11^m) star gives a spacecraft attitude determination on the 1 to 2 mas (5 to 10 nrad) level, much better than required. The amount of image smearing is not critical. A large number of such observations (30 per minute) can be accommodated with less than 1% of the down–link data rate.

1 Introduction

The main goal of this investigation is to come up with an answer for "how good can we determine the spacecraft attitude". Thus the precision of the cross–scan centering of stellar images need to be estimated. This is obtained from evaluating 2–dimensional "postage stamps" data extracted for certain stars. The *accuracy* of the attitude determinations is believed to be not much worse than the *precisions* presented here, because random noise will dominate such single observations. It is assumed, that (at least in the long run) all significant systematic errors are taken out. This is relatively easy here, because systematic errors on the 300 μas level are already not significant for a single attitude determination. Results are obtained from simulations with realistic noise and image smearing. The general centering *accuracy* as part of the error budget over the entire mission is a totally different story, not dealt with here.

2 Input Data

The following simulations are based on PSF data obtained from Scott Horner. For a given wavelength (350 nm to 950 nm, step = 10 nm) 2–dimensional image intensities are given for 1/1000 pixel steps along scan (12 pixel wide) and 1/2 pixel steps for cross–scan (20 pixel wide). These intensities are already normalized with respect to the integral of a single PSF. I also used Scott's throughput file for the instrumental throughput = $f(\lambda)$, which includes the quantum efficiency of the CCD.

3 Basic characteristics of simulations

- on the optical axis only
- rectangular aperture mainly, with a C–shaped comparison
- poly–chromatic main sequence stars (black body approximation)
- 2–dim "postage stamp", 9 (scan) by 15 (cross–scan) pixels
- realistic CCD noise (Poisson noise; read out noise = 8 electrons)
- including image smearing, both axis, various cases
- round to 12–bit ADU integers, full well = 100,000, gain=25

4 Procedure details

- generate PSF's of stars using sum of 13 monochromatic PSF's (350 to 950, step=50 nm) with weights from black body distribution
- apply image smear symmetric to original center position, integrate along the path in 1/100 pixel along scan steps, simultaneous in both axes, interpolation for cross-scan steps,
- offset resulting image, maximal ±0.5 pixel, for individual simulations in groups of fixed sub–pixel location
- run about 50 simulations per case
- fit with 2–dim *elliptical* Gaussian, allow axes orientation to be *rotated* with respect to x,y axes, thus use a 7 parameter model (amplitude, background, center posit. x,y, profile width major, minor axis, and rotation parameter)
- the standard errors in the positions are obtained from the scatter of the fit positions minus "true" center positions minus offset from systematic error as function of the sub–pixel location of the image center

Note a) The fit model adopted here does not match the PSF perfectly. However, the model approximates the central peak of the PSF's sufficiently and the Gaussian model is a robust estimator. Due to simultaneous smearing along both axes the orientation of the resulting PSF is rotated in the focal plane, thus the major axis of the quasi elliptical image profile is no longer parallel to the cross–scan direction.

Note b) The black body approximation used here is good enough. Even the difference between an O5 and M5 star is ≈ 10% in the *fit precision*. The shape of the smeared, poly–chromatic PSF's look very much the same, mainly the profile width varies as a function of the spectral type. The profile width is one of the fit parameters. Real data reductions will have to determine *systematic* position offsets as a function of spectral type as well as sub–pixel location and other things first before the random noise statistics of this investigation applies.

5 Cases

The main results presented below are for an **F0** main sequence star (T_{eff} = 7300 K). For comparison some simulations were run also for an O5 and M5 main sequence star. The following image smearing cases are considered, with x being along scan, and y in cross–scan direction; unit = pixel. The approximate full width at half maximum (FWHM) for the marginal distributions along x and y for images of all 3 spectral type stars are given as well:

case	smear		:	FWHM O5		:	FWHM F0		:	FWHM M5	
number	x	y	:	x	y	:	x	y	:	x	y
1	0.0	0.0	:	0.7	4.2	:	0.8	4.6	:	1.0	5.9
2	1.0	0.0	:			:	1.1	4.6	:		
3	1.0	3.0	:			:	1.2	5.3	:		
4	1.0	6.0	:	1.1	6.8	:	1.2	6.8	:	1.3	7.5

6 Saturation and magnitudes

Saturation here means the amplitude of a profile is 100,000 electrons or 4000 ADU's. This is as bright as a star can be in order not to exceed the full well capacity of any CCD pixel; with the worst case having the peak of the profile falling on the center of a pixel. Depending on the amount of smearing, this corresponds to slightly different magnitudes (total flux) of a star because of the wider PSF's. The following table gives the flux in 1000 electrons (sum over all pixels of an image) which a star will have in order to reach saturation in the central pixel. Also given is the corresponding magnitude differences (Δm) relative to the not smeared F0 star. *C* means C–shaped aperture, *rect* mean rectangular aperture.

aperture	spectr. type	smear case	flux k e^-	Δm mag
rect	F0	1	465	0.00
		2	660	0.38
		3	708	0.45
		4	838	0.64
C	F0	1	269	−0.59
		2	420	−0.11
		3	555	0.19
		4	848	0.65
rect	O5	1	359	−0.28
		4	720	0.48
rect	M5	1	707	0.46
		4	1070	0.91
rect	A0	1	425	−0.10

A positive Δm means the star needs to be brighter by that amount than the reference case (F0, no smear, rectangular aperture) to reach the same peak (saturation) amplitude.

The last line in the table helps to relate the relative magnitudes to Scott's absolute magnitudes. According to Scott there are enough photons for an A0 star to just saturate at magnitude 9.0, thus reaching our goal with the rectangular aperture.

7 Results

The following table gives the fit precision (standard error) of the along scan (σ_x) and cross–scan (σ_y) positions for an F0 star in units of pixel based on the **rectangular** aperture for the smearing cases mentioned above. Individual numbers in the table should be good to about 10% relative to each other.

sat = saturation magnitude (brightest "good" star)
+1.0 = 1.0 magnitude fainter than saturation ...
ADU = amplitude in analog to digital units = DN's = CCD output numbers

mag	ampl.	along scan σ_x				:	cross–scan σ_y			
	ADU	case 1	case 2	case 3	case 4	:	case 1	case 2	case 3	case 4
sat	4000	0.0008	0.0008	0.0009	0.0009	:	0.0035	0.0035	0.0040	0.0042
+1.0	1600	0.0016	0.0015	0.0014	0.0012	:	0.0047			
+2.0	630	0.0021	0.0021	0.0020	0.0019	:	0.0077	0.0072	0.0095	0.0096
+3.0	250	0.0035	0.0033	0.0035	0.0032	:	0.0130			
+4.0	100	0.0057	0.0055	0.0055	0.0050	:	0.0185	0.0200	0.0200	0.0250
+5.0	40	0.0110	0.0110	0.0100	0.0090	:	0.0400			
+6.0	16	0.0170	0.0180	0.0180	0.0150	:	0.0600	0.0590	0.0550	0.0760
+6.5	10	0.0250			0.0250	:	0.1000			

The following table gives results only for **case 4** for other spectral types with the same rectangular aperture as before, as well as for the C–shaped aperture with the F0 star.

mag	ampl.	O5 rectan.		:	M5 rectan.		:	F0 C-shape	
	ADU	σ_x	σ_y	:	σ_x	σ_y	:	σ_x	σ_y
sat	4000	0.0008	0.0041	:	0.0008	0.0042	:	0.0010	0.0040
+2.0	630	0.0018	0.0110	:	0.0018	0.0100	:	0.0022	0.0100
+4.0	100	0.0050	0.0250	:	0.0050	0.0290	:	0.0050	0.0250
+6.0	16	0.0150	0.0690	:	0.0140	0.0780	:	0.0200	0.0790

8 Discussion

The errors for the along scan direction are very low (1/1000 pixel) for bright stars. This can be understood by the image elongation in the cross–scan direction, which provides quasi multiple determinations of the x–coordinate from a single image from various $y = const$ lines. However, with real data, which will include off–axis images and some image asymmetry, this extreme precision will be degraded to some extend.

The cross–scan fit precision is almost independent of the image smearing, providing a sufficiently large window is used. With 200 mas per pixel and a standard error of ≤ 0.01 pixel for stars in the magnitude range from saturation to 2 mags fainter, this will give an attitude determination good to 2.0 mas (10 nrad) from a single observation! The brightest good stars will give 0.8 mas (4 nrad) and even stars 3^m fainter as saturation will give less than 3.0 mas (15 nrad).

There is no significant variation with the spectral type of the stars. However, an M5 star needs to be brighter than an F0 star to reach saturation due to the broader PSF. Results from the C–shaped aperture are slightly worse.

9 Conclusions

Attitude determination: can be obtained instantaneously from a single 2–dim observation of a bright star ($\approx$ 9 to 11 mag) with a standard error of about 1 to 2 mas (5 to 10 nrad).

Off the optical axis the precisions for cross–scan positions are expected to be slightly worse. Systematic errors can likely be modeled to an extend not affecting the overall accuracy of a single observation significantly. Thus the errors given here should be close to the external errors expected from the mission.

The effects of CCD cosmetics and radiation damage are yet unknown. Gathering cross–scan data from different CCD's is advisable to average out such effects on the attitude determination.

Limits on image smearing are not set by the 2–dim observations for attitude determination. A wide range of 0 to 6 pixel image smearing for the cross–scan direction is acceptable here. However, the effect of image smearing on *systematic errors* in normal, marginal distribution observations and otherwise is not investigated here.

Frequent observing: is strongly recommended to get such "postage stamps" as often as reasonable. The data from such a single 2–dim map are less than 20 times as large as for the 1–dim marginal profiles of a single normal observation. Assuming $\leq 1\%$ of the total down–link data to be from 2–dim maps, this means one star out of 2000. With about 1000 normal stars per second this means we could get a 2–dim map every 2 seconds. This will give us about 30 attitude measures over 1 minute, obtained from various CCD's over the field of view, which is desirable to have as a consistency check. Even if we do not need an attitude determination that often, it will be of great help as "sanity check" and for diagnostics in case something goes wrong. It would be easy for instance to empirically verify that the attitude runs smoothly on time scales of a minute and any unexpected drift could be observed directly.

The Input Catalog should contain a flag to pre–select such 2–dim map stars. Selection should be made by magnitude (function of spectral type) and possible contamination by other nearby stars (avoiding known double stars). They also should be distributed evenly over the sky. With over 10 stars per square degree in the 9 to 11 mag range in most areas of the sky, there are enough such map stars to choose from.

If possible, science target stars should be included in this list. There is a potential for a significantly better along scan coordinate determination from these 2–dim maps than from marginal 1–dim data of normal observations. How much impact this will have on the mission–overall accuracy of those stars remains to be seen, however, individual observations will have a very high precision. This will be of advantage in particular for stars with suspected planets around them, giving higher than normal individual epoch observations for detecting small masses with relatively small orbital periods.