Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

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To: Distribution
From: R.D. Reasenberg
Subject: Precession of the FAME spacecraft

I. Introduction.

FAME is a full-sky astrometric survey instrument with a nominal mission accuracy of 50 μas for bright stars. If flown as presently conceived, FAME will be the first spacecraft to use solar radiation pressure to provide the drive needed to change the instrument pointing direction as required by the observing schedule. It will not, however, be the first spacecraft to make use of the torque available from solar radiation pressure. The GOES-8 and GOES-9 spacecraft each had a "trim tab" that was adjusted by ground command to provide solar radiation pressure torque to reduce the need for active control of angular momentum and conserve propellent (Harvie et al. 1996).

We assume here that the spacecraft is spinning and that the spin direction precesses around the Sun direction under the torque due to solar radiation pressure acting on the circular shield. See Fig. 1 in TM97-03 (Reasenberg 1997), where the use of radiation pressure precession is introduced and analyzed. The purpose of this memorandum is to incorporate in the analysis of the rotation dynamics the effect of the change of direction of the solar radiation that comes with the Earth's motion around the Sun.

The angle $\bar{\xi}$, between the nominal spacecraft spin axis and the Sun direction, is bounded at the high end by the Sun avoidance requirement and at the low end by the need for observational diversity, which reduces estimator degeneracy. For present purposes, the mean value will be assumed to be about 45 deg. However, it will be shown that this angle will vary over the precession cycle, and that the degree of variation is inversely proportional to the precession rate.

In section II, we find the equations of rotational motion of the spacecraft, which is treated as a heavy symmetrical fast top. This is a new celestial mechanics problem. For simplicity, we treat the motion of the angular momentum, not that of the body-fixed axes. Further, the distinction between the normal to the shield and the direction of the angular momentum vector is ignored in calculating the radiation pressure torque. It will be seen that this is a "higher order" correction. In Section III, a solution to the equations of motion is postulated, and its free parameters are found. The parameters are represented as power series in a small quantity, and only the first small term is kept in each case.

In Section IV, we examine the results of a numerical integration of the equations of motion. These confirm the analytic solution and provide an approximation to the next higher term in the series expansion of the precession rate. This is followed by a concluding discussion in section V.

II. Equations of Motion

We consider a (rotating) right-handed coordinate system (x, y, z) with the z axis pointing toward the north ecliptic pole. The x axis, which points toward the Sun, is assumed to move at a uniform rate in the ecliptic (i.e., the eccentricity of Earth's orbit is ignored).

The spacecraft has a nominal spin rate of three rotations per hour and nominal precession rate of one cycle per ten days. Thus, we are dealing with a "fast top" in that the angular momentum vector undergoes a small change of direction during a single rotation (about 0.35 deg). (Goldstein, p. 169) To properly model the FAME observable, we will need to know the complete motion of the instrument, most conveniently represented by the spin vector (e.g., the Euler angles describing the orientation of the body-fixed axes, including the nominal spin axis), as a function of time. However, here we consider the motion of the spacecraft angular momentum, rather than analyzing the motion of the spin axis directly by means of the Euler equations. This approach greatly simplifies the problem and facilitates the desired physical description. The determination of the spin vector from the motion of the angular momentum vector is straight forward, but not addressed here. For the nominal spin and precession rates, the angular momentum and nominal spin axis are separated by about 10^{-3} radian, or about 3.3 arc min. It will be seen that this is small compared to the cyclic deviation of the angular momentum vector from a smooth, uniform rotation around the Sun direction.

In the rotating coordinate frame, the equation of motion of the angular momentum, L, is

$$N_R = N_I = \left(\frac{dL}{dt}\right)_I = \left(\frac{dL}{dt}\right)_R + \omega \times L \qquad \textbf{(1)}$$

where N is the external torque, the subscripts R and I refer to the rotating and inertial frames respectively, and ω is the angular velocity of the rotating frame. Let ξ be the angle between the spacecraft angular momentum vector and the x axis (Sun direction), and ν be the angle between the ecliptic and the plane containing the angular momentum vector and the x axis. See Fig. 1. Then

$$\hat{L} = (\cos\xi, \sin\xi\cos\nu, \sin\xi\sin\nu) \qquad \textbf{(2)}$$

and

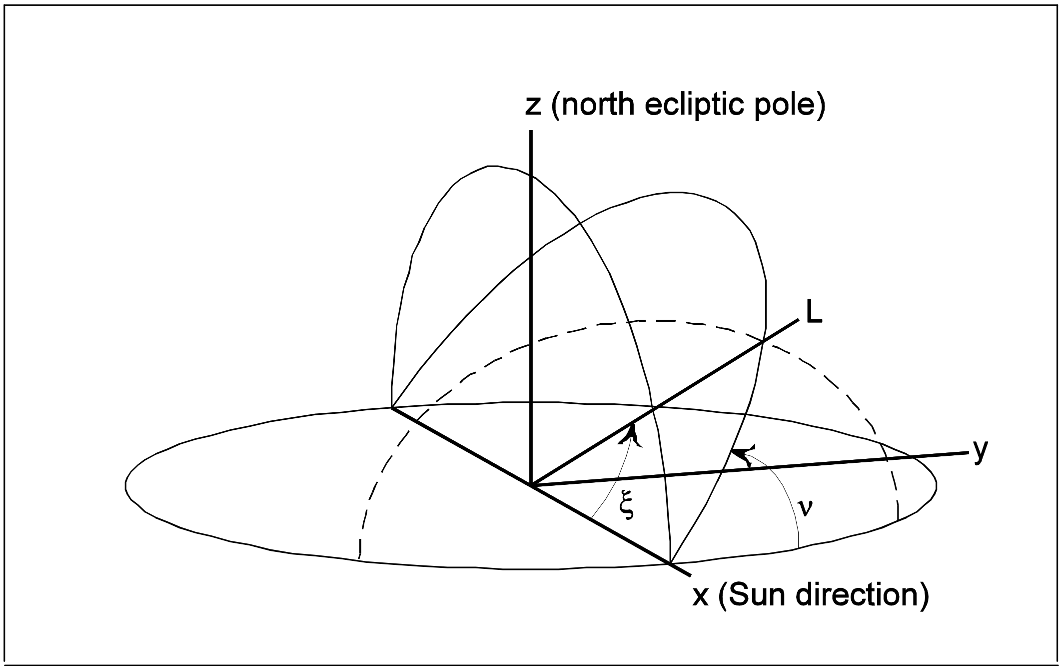

Figure 1. Spherical geometry of precession with ξ = 45 deg and ν = 60 deg. The dashed line shows the trajectory of the spacecraft angular momentum vector on the celestial sphere. The view is from 15 deg above the reference plane and 20 deg to the left of the x axis.

$$\omega \times \hat{L} = |\omega| \, (-\sin\xi \cos\nu, \, \cos\xi, \, 0) \qquad \textbf{(3)}$$

It was shown in TM97-03 that the torque due to solar radiation pressure is proportional to $\sin\bar{\xi}\cos\bar{\xi}$. However, here we approximate this factor as $\sin\xi\cos\xi$, so that

$$N = N_0 \sin\xi \cos\xi \, (0, \, -\sin\nu, \, \cos\nu) \qquad \textbf{(4)}$$

where N_0 depends on the size, shape, and optical properties of the solar shield.[1] For convenience, we introduce the parameter $A = N_0 / |L|$, where |L| is the angular momentum of the spacecraft and A is a scale factor (with dimension = 1/time) between torque and angular momentum. The parameter A thus provides a time scale for precession. (Note that, because the shield is centered and optically uniform, $\hat{L} \cdot N = 0$, which implies that |L| is constant.) By combining the above equations, we obtain the three components of the equations of motion. The x and y components are

[1] The effect of failing to distinguish between ξ and $\bar{\xi}$ in the calculation of torque is a few parts per million in torque at ξ_0 = 45 deg. This is dwarfed by physical effects that we will need to include in the analysis of the FAME data.

$$\sin\xi\,\dot{\xi} = -|\omega|\sin\xi\,\cos\nu \qquad \textbf{(5)}$$

$$\sin\xi\;\sin\nu\;\dot{\nu} - \cos\xi\;\cos\nu\;\dot{\xi} = A\,\sin\xi\;\cos\xi\;\sin\nu + |\omega|\cos\xi \qquad \textbf{(6)}$$

and the z component is redundant. From Eq. 5, we obtain

$$\dot{\xi} = -|\omega|\cos\nu \qquad \textbf{(7)}$$

which, when applied to Eq. 6, yields

$$\dot{\nu} = A\,\cos\xi + |\omega|\,\frac{\cos\xi\,\sin\nu}{\sin\xi} \qquad \textbf{(8)}$$

Finally, we introduce the small dimensionless parameter $\alpha = |\omega|/A$, which will serve as the expansion parameter for the perturbation expansion of the equations of motion below.

$$\dot{\xi} = -A\,\alpha\cos\nu \qquad \textbf{(9)}$$

$$\dot{\nu} = A\,\cos\xi\left(1 + \alpha\,\frac{\sin\nu}{\sin\xi}\right) \qquad \textbf{(10)}$$

Anticipating the results of the next section, we find that for ξ_0 = 45 deg and a nominal ten day precession period, α = 0.0194. We will find that the motion can be described in terms of functions that contain power series in α.

III. Solving the Equations of Motion

Equations 9 and 10 are easily solved for α = 0, i.e., ignoring the motion of the Sun. By inspection, we find

$$\begin{aligned} \xi &= \xi_0 \\ \nu &= \nu_0 + \Omega\,t \end{aligned} \qquad \textbf{(11)}$$

where $\Omega = A\cos\xi_0$, ξ_0 is a given, and ν_0 provides an arbitrary initial phase at t = 0. To find a better approximation, we introduce a trial solution[2] with five (four new) parameters to be determined (Ω, ξ_1, ν_1, ϵ_ξ, and ϵ_ν):

$$\xi = \xi_0 - \xi_1 \sin(\Omega t + \epsilon_\xi) \tag{12}$$

$$\nu = \nu_0 + \Omega t - \nu_1 \cos(\Omega t + \epsilon_\nu) \tag{13}$$

In the above, ϵ_ξ and ϵ_ν provide independent phases for the perturbation terms (ξ_1 and ν_1 respectively).

To insert Eqs. 12 and 13 into Eqs. 9 and 10, we will need the following expansions in Bessel functions of the first kind:

$$\begin{aligned}
\cos\xi &= \cos\xi_0 [J_0(\xi_1) + 2J_2(\xi_1)\cos(2\Omega t + 2\epsilon_\xi)] + 2\sin\xi_0\, J_1(\xi_1)\sin(\Omega t + \epsilon_\xi) + \ldots \\
\sin\xi &= \sin\xi_0 [J_0(\xi_1) + 2J_2(\xi_1)\cos(2\Omega t + 2\epsilon_\xi)] - 2\cos\xi_0\, J_1(\xi_1)\sin(\Omega t + \epsilon_\xi) + \ldots \\
\cos\nu &= \cos(\nu_0 + \Omega t)[J_0(\nu_1) - 2J_2(\nu_1)\cos(2\Omega t + 2\epsilon_\nu)] \\
&\quad + 2\sin(\nu_0 + \Omega t)J_1(\nu_1)\cos(\Omega t + \epsilon_\nu) + \ldots \\
\sin\nu &= \sin(\nu_0 + \Omega t)[J_0(\nu_1) - 2J_2(\nu_1)\cos(2\Omega t + 2\epsilon_\nu)] \\
&\quad - 2\cos(\nu_0 + \Omega t)J_1(\nu_1)\cos(\Omega t + \epsilon_\nu) + \ldots
\end{aligned} \tag{14}$$

The required series expansions of the Bessel functions *per se* are given by:

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[2] There is a heuristic reason for the form of the trial solution. Looking toward the Sun from the spacecraft, with the north ecliptic pole called up, the Sun is seen (in an inertial frame) to move to the left as a result of Earth's annual motion. Assume the spin vector will precess clockwise around the Sun in the rotating coordinate system that has the Sun along the x axis. As the spin vector is rising through the ecliptic (to the left of the Sun), the Sun is moving to decrease ξ: $\dot{\xi}$ is at a minimum. Next consider the spin vector at its maximum height above the ecliptic. It is moving to the right as the Sun moves to the left: $\dot{\nu}$ is at a maximum. These results are consistent with the trial solution if ξ_1 and ν_1 are positive and ϵ_ξ and ϵ_ν are approximately equal to ν_0, as will be shown below to be one solution.

$$\cdot\, J_0(w) = 1 - \frac{w^2}{4} + \frac{w^4}{64}$$

$$J_1(w) = \frac{w}{2} - \frac{w^3}{16} \tag{15}$$

$$J_2(w) = \frac{w^2}{8} - \frac{w^4}{96}$$

(Olver 1964, see Eqs. 9.1.42 - 9.1.45 and 9.1.10). Then, by differentiating Eq. 13 and applying Eq. 14 to Eq. 10, keeping terms to lowest order in small quantities (α, ξ_1, and ν_1), we get

$$\dot{\nu} = \Omega + \nu_1 \Omega \sin(\Omega t + \epsilon_\nu) =$$

$$A\left(\cos\xi_0 + 2\sin\xi_0\, J_1(\xi_1)\sin(\Omega t + \epsilon_\xi)\right)\left(1 + \alpha\,\frac{\sin(\nu_0 + \Omega t)}{\sin\xi_0}\right) \tag{16}$$

By expanding and collecting terms in Eq. 16 that are respectively constant in time, proportional to $\cos(\Omega t)$, and proportional to $\sin(\Omega t)$, and keeping only terms that are first order in small quantities, we obtain

$$\Omega = A\cos\xi_0 \tag{17}$$

$$\nu_1\,\Omega\sin(\epsilon_\nu) = A\,\alpha\cos\xi_0\,\frac{\sin\nu_0}{\sin\xi_0} + 2\,A\sin\xi_0\,J_1(\xi_1)\sin(\epsilon_\xi) \tag{18}$$

$$\nu_1\,\Omega\cos(\epsilon_\nu) = A\,\alpha\cos\xi_0\,\frac{\cos\nu_0}{\sin\xi_0} + 2\,A\sin(\xi_0)\,J_1(\xi_1)\cos(\epsilon_\xi) \tag{19}$$

By applying Eq. 17 to Eqs. 18 and 19 and collecting terms, we get, respectively

$$\begin{aligned} \nu_1\sin(\epsilon_\nu) &= Q\sin(\nu_0) + R\sin(\epsilon_\xi) \\ \nu_1\cos(\epsilon_\nu) &= Q\cos(\nu_0) + R\cos(\epsilon_\xi) \end{aligned} \tag{20}$$

where

$$Q = \frac{\alpha}{\sin(\xi_0)} \tag{21}$$

and

$$R = \frac{2\sin(\xi_0)\, J_1(\xi_1)}{\cos(\xi_0)} \approx \frac{\sin(\xi_0)\, \xi_1}{\cos(\xi_0)} \tag{22}$$

We next perform a variant of the above procedure by differentiating Eq. 12 and applying Eq. 14 to Eq. 9. Again, we keep terms to lowest order in small quantities (α, ξ_1, and ν_1).

$$\begin{gathered} \dot{\xi} = -\xi_1 \Omega \cos(\Omega t + \epsilon_\xi) = \\ -A\alpha\left(\cos(\nu_0 + \Omega t) + 2\sin(\nu_0 + \Omega t) J_1(\nu_1) \cos(\Omega t + \epsilon_\nu)\right) \end{gathered} \tag{23}$$

By applying Eq. 17, expanding and collecting terms that are respectively proportional to $\cdot\cos(\Omega t)$ and proportional to $\sin(\Omega t)$, and keeping only terms that are first order in small quantities, we obtain

$$\begin{gathered} \xi_1 \cos(\xi_0) \cos(\epsilon_\xi) = \alpha \cos(\nu_0) \\ \xi_1 \cos(\xi_0) \sin(\epsilon_\xi) = \alpha \sin(\nu_0) \end{gathered} \tag{24}$$

To simplify the above equations, we introduce

$$K = \frac{\cos(\xi_0)\, \xi_1}{\alpha} \tag{25}$$

and obtain

$$\begin{gathered} \cos(\nu_0) = K \cos(\epsilon_\xi) \\ \sin(\nu_0) = K \sin(\epsilon_\xi) \end{gathered} \tag{26}$$

Recall that ϵ_ξ and ϵ_ν were introduced to provide independent phases for the perturbations terms (ξ_1, and ν_1 respectively). We expect ϵ_ξ and ϵ_ν to be connected to ν_0 in a simple way. If we take the ratio of the above equations, then we find that ν_0 and ϵ_ξ differ by 0 or π. It follows that

$$K = \pm 1$$
$$\xi_1 = \pm \frac{\alpha}{\cos(\xi_0)} \tag{27}$$
$$\epsilon_\xi = \nu_0 + \pi/2 \mp \pi/2$$

When these are applied to Eq. 12, we obtain a result independent of the sign of K

$$\xi = \xi_0 \mp \frac{\alpha}{\cos(\xi_0)} \sin(\Omega t + \nu_0 + \pi/2 \mp \pi/2) = \xi_0 - \frac{\alpha}{\cos(\xi_0)} \sin(\Omega t + \nu_0) \tag{28}$$

Note that for the FAME nominal of P = 10 days, $|\xi_1| = |\omega|/\Omega = P/(1\text{ year}) = 0.02738$, where P is the precession period.

We return now to Eq. 20 and apply Eq. 27 to obtain for the K = 1 case

$$\nu_1 \sin(\epsilon_\nu) = (Q + R)\sin(\nu_0)$$
$$\nu_1 \cos(\epsilon_\nu) = (Q + R)\cos(\nu_0) \tag{29}$$

Following the logic used above, we introduce $G = (Q + R)/\nu_1$ and find that G = ± 1, $\epsilon_\nu = \nu_0 + \pi/2 \mp \pi/2$, and

$$\nu_1 = \frac{\pm\alpha}{\sin(\xi_0)\cos^2(\xi_0)} \tag{30}$$

When these results are applied to Eq. 13, we obtain

$$\nu = \nu_0 + \Omega t - \frac{\alpha}{\sin(\xi_0)\cos^2(\xi_0)} \cos(\nu_0 + \Omega t) \tag{31}$$

It is easily shown that when the above analysis is repeated for K = -1, the same result is obtained. Thus, for the FAME nominal of ξ_0 = 45 deg, we find that $\nu_1 = 2\,\xi_1$.

The next step is to obtain an expression for Ω to higher (i.e., second) order in α. This requires repeating the derivation of Eq. 17, but to higher order in small quantities (α, ν_1, and ξ_1). To do this, we first use Eqs. 14 and 15 and find

$$\begin{aligned} \frac{1}{\sin\xi} &= \frac{1}{\sin\xi_0}\left(1 + \xi_1\frac{\cos\xi_0}{\sin\xi_0}\cos(\Omega t + \epsilon_\xi)\right) \\ &= \frac{1}{\sin\xi_0}\left(1 + \frac{\alpha}{\sin\xi_0}\cos(\Omega t + \nu_0)\right) \end{aligned} \tag{32}$$

after applying Eq. 27. (Note that it later becomes apparent that we need the last expression only to lowest order.) Next, we differentiate Eq. 13, and insert Eq. 10. After applying the above and Eq. 14 and replacing both ϵ_ν and ϵ_ξ by ν_0, we obtain

$$\begin{aligned} \dot{\nu} = \Omega + \nu_1\Omega\sin(\Omega t + \nu_0) &= A\,[\cos\xi_0 J_0(\xi_1) + 2\sin\xi_0 J_1(\xi_1)\cos(\Omega t + \nu_0)] \\ \times\left(1 + \alpha\,[\sin(\Omega t + \nu_0)J_0(\nu_1) - 2\cos(\Omega t + \nu_0)^2 J_1(\nu_1)]\right.&\left.\frac{1}{\sin\xi_0}\left(1 + \frac{\alpha}{\sin\xi_0}\cos(\Omega t + \nu_0)\right)\right) \end{aligned} \tag{33}$$

If we expand Eq. 33 with the help of Eq. 15, and collect the time-independent parts up to α^2, then we find that

$$\Omega = A\cos(\xi_0)\left(1 - \frac{\xi_1^2}{4} + \frac{\xi_1\alpha}{2\cos(\xi_0)}\right) \tag{34}$$

which, with the help of Eq. 27, reduces to

$$\begin{aligned} \Omega &= A\cos\xi_0(1 - \kappa\alpha^2) \\ \kappa &= \frac{1}{4\cos^2(\xi_0)} \end{aligned} \tag{35}$$

For the FAME nominal of $\xi_0 = 45$ deg, we find that $\kappa = 0.5$.

Recall that in Section II we defined two parameters: (1) $A = N_0 / |L|$, where $|L|$ is the angular momentum of the spacecraft and N_0 depends on the size, shape, and optical properties of the solar shield; and (2) $\alpha = |\omega| / A$, where ω is the angular velocity of the rotating frame, i.e., the mean orbital rate of Earth. Using the definition of α, we can rewrite Eqs. 28, 31, and 35

$$\xi = \xi_0 - \frac{|\omega|}{A\cos(\xi_0)} \sin(\Omega t + \nu_0) \tag{36}$$

$$\nu = \nu_0 + \Omega t - \frac{|\omega|}{A\sin(\xi_0)\cos^2(\xi_0)} \cos(\Omega t + \nu_0) \tag{37}$$

$$\Omega = A\cos(\xi_0)\left(1 - \kappa \frac{|\omega|^2}{A^2}\right) \tag{38}$$

Written this way, the equations show that fast precession is relatively simple and well represented by an adiabatic approximation. Slow precession has a more eccentric form. As α approaches 1, the analysis given above breaks down and other forms of the solution are needed.

IV. Results of Numerical Integration.

In order to confirm and expand upon the analytic results above, I have used Mathematica[3] to numerically integrate the equations of motion for ξ_0 = 45 deg. The initial conditions for the integration are

$$\begin{aligned} \xi(\text{initial}) &= \xi_0 \\ \nu(\text{initial}) &= -\nu_1(\alpha) \end{aligned} \tag{39}$$

Equation 30 provides ν_1 for $\alpha = 0$. The correct second initial condition must be obtained by an iterative procedure, starting with nominal initial conditions (ξ_0 and $-\nu_1$):

[3] The reference to a commercial product is for technical communication only, and does not constitute an endorsement of the product.

Integrate equations of motion.
Get Ω from $\xi(t)$ by finding the period of its time variation.
Get $\nu_1(\alpha)$ from $\nu(t) - \Omega t$ by finding the amplitude of its time variation.

This sequence converges in three to five iterations for $\alpha = \{0.01 \ldots 0.06\}$. Following convergence, one can find correct values of ν_1, ξ_1, and Ω for a particular value of α.

After setting the "accuracy goal" for the integration to10^{-13}, I made a series of runs with $\alpha = \{0.01, 0.02 \ldots 0.06\}$ to find the precession period, $P = 2\pi/\Omega$. From Eq. 35, we expect

$$P = P_0(1 - \kappa\alpha^2) \tag{40}$$

A plot of κ against α suggested a parabola with extreme between $\alpha = 0.01$ and $\alpha = 0.02$. The three-parameter fit yielded $\kappa = -0.500050 - 0.00703\,\alpha + 0.32777\,\alpha^2$, and $\kappa(0.02) = -0.500060$. The fit had an rms residual of $7.5\ 10^{-6}$, and the minimum is -0.500088 at $\alpha = 0.011$.

I followed a similar procedure for the other two parameters to obtain $\xi_1(\alpha) = -\sqrt{2}\ \alpha\ (0.99998 - 2.3398\,\alpha^2)$, and $\nu_1(\alpha) = 2\sqrt{2}\ \alpha\ (1.00002 - 1.54885\,\alpha^2)$. The rms residual error was, respectively, $1.6\ 10^{-5}$, and $2.2\ 10^{-5}$.

V. Discussion.

This memorandum introduces a new celestial mechanics problem and its lowest-order analytic solution. By comparison with results I obtained by numerically integrating the equations of motion, I have determined that the analytic expressions for ξ_1, ν_1, and Ω are reasonable.

Eventually, it is likely that a more refined analysis will be performed, and that this will require computer algebra to solve the equations of motion to higher order in α and to include the effect of the Earth's orbit eccentricity. There may be interesting effects when the precession period is an exact multiple of a year. Long term, this could produce a "pumping" of the precessional motion, leading to a long period variation of ξ, for example. However, neither extension is likely to be of importance for the FAME mission.

Consider the numerically determined corrections to the precession period for the nominal $\alpha = 0.02$. These terms depend on the third and fourth powers of α. Assume that a coherent span of data covers 36 spacecraft rotations or 12 hours. At the end of that period, each correction term contributes about 0.004 arc sec to the precession phase. (Incidentally, note that the contributions are of opposite sign, and cancel by an order.) The contribution from ν depends on the third power of α and, at the end of 12 hours, is 2.3 arc sec of precession phase. Finally, the correction to ξ_0, if not considered, would cause the estimate of ξ_0 from the data to be off by 5.4 arc sec. This would map into a miscalculated Ω that would cause an error in precession phase of 1.7 arc sec. However, precession rate, Ω, would be determined from the observations directly and thus this error would not be seen numerically. Assuming the next term in each case is smaller by a single factor of α, then the present analytic description would be nearly good enough to use for the data analysis. It would, however, be imprudent to use expressions that are "nearly good enough" given that one can easily do much better and eliminate a potential source of systematic error.

In the analysis of the FAME mission data, we will require a high precision model of the spacecraft rotation. This will serve as the basis for the Spiral Reduction. Such a model is likely to be based on the numerical integration of an enhanced set of differential equations, and the corresponding set of variational equations, as would be done in a trajectory estimation problem. It remains to be seen whether it will be necessary to integrate the full set of (six) equations or whether two will be sufficient. In either case, we may introduce into the model some *ad hoc* perturbations that we will assume to be so small (i.e., at the arc second level) that they need not be numerically integrated. These would include the nutation that is excited, for example, by the torque due to Earth radiation entering the view ports or thermal radiators on the bus. (The nutation of the spacecraft needs to be investigated, but is beyond the scope of the present work.)

Recent preliminary analysis of the effect of fluctuations of the solar output suggest that it may be desirable to measure the solar output in each of a few optical bands and to find a linear combination of these measurements that represents the momentum absorbed by the shield. This could be normalized to yield the "solar torque factor." If this approach is taken, then it will be useful to numerically integrate the full set of six equations and to include in the integration the time-varying solar torque factor.

A high rate of precession helps to ensure uniform sky coverage, but decreases the number of observations of a given star per visit[4]. The nominal precession rate of $\Delta\nu = 0.5$ deg per rotation was set to keep the star images from moving across too many CCD columns as they move along the column direction. This is a signal-to-noise issue for faint stars and not a precise requirement. Thus, if the precession rate were to vary in a known way by 10%, there would be no harm done. In fact, we have not "optimized" the precession rate, and I suspect that when we do we will find that considerable variation is acceptable as long as the change in Ω is slow and predictable.

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[4] As the spacecraft scans the celestial sphere, a given star will be observed in each of several successive rotations and then not observed for some time. This sequence of observations is called a visit. With the nominal FAME parameters, the duration of a visit ranges from 6 to 90 rotations, with an average of just over four observations per rotation. For further discussion, see Reasenberg (SAO-TM-98-07).

VI. References.

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VII. Acknowledgments.

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VII. Distribution

SAO:

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