Harvard-Smithsonian Center for Astrophysics
Precision Astronomy Group
60 Garden St., M/S 63, Cambridge, MA 02138

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From: J.F. Chandler and R.D. Reasenberg
Subject: GAMES sensitivity III. Effect of fast rotation

I. INTRODUCTION

This memorandum is the third in a series investigating the dependence of GAMES performance on a variety of design parameters affecting the observing geometry. These studies all consider the first stage of data analysis (modeling the spacecraft rotation from "batches" of astrometric data) and rate the instrument's performance by how well it ties together the swath of sky seen during a "batch interval" (of order one day). The first memorandum (Chandler and Reasenberg 1998a, hereafter TM98-01) described the effect of varying the complexity of the rotation model and concluded that avoiding frequent attitude corrections ("rotation breaks") is an important consideration in increasing the scientific output of the mission. The second memorandum (Chandler and Reasenberg 1998b, hereafter TM98-02) examined the effect of varying the angle between the instrument's two fields of view (the "basic" or "opening" angle).

The present study re-examines the questions of the first two memoranda with two new variations: a wider field of view and faster rotation. It addresses the effect of a larger number of rotations in a batch interval. The remainder of this memorandum describes the configurations used in this study (Section II), presents the results from a series of simulations (Section III), and draws conclusions about spacecraft design considerations (Section IV). A glossary of terms is again included as Appendix A.

II. OPERATIONAL MODEL

Refer to TM98-01, Sections II - IV, for a description of the spacecraft model and of the methods used for simulating observations and reducing the simulated data. Although the present study is a continuation of the previous two, it differs in several details. First, the width of the field of view has been increased from 0.75 to 1.6 deg, in accordance with recent changes in the optical design concepts (Phillips 1998). This increase leads to a larger minimum overlap of the observing spiral band between successive instrument rotations (75% on average instead of 50%). As a result, each star seen by the instrument will be observed on four or more successive rotations (except for some stars seen at the beginning or end of the batch interval). In addition, the rotation rate has been increased by a factor of four, so that the rotation period has dropped from 2 hr to 30 min, in keeping with an alternate scheme for the spacecraft design (Reasenberg and Phillips 1998). At the same time, the precession rate has been raised by the same factor from the value used in TM98-02 (and in some of the runs in

TM98-01). Again, as in TM98-02, only a single value of the spacecraft precession rate is used in this study, i.e., 24 deg/d (= 0.5 deg/spacecraft rotation). By keeping the same ratio between the rotation and precession rates, we have avoided making any further alteration in the spiral overlap; we have also kept the same level of precession-induced variation in the instantaneous scan direction of star images on the detectors relative to the average scan direction (which is intended to align with the columns of the CCD detectors).

We have also scaled the average lengths of the rotation spans, so that the "short" average length is now 5 min, and the "long" average length is 30 min (still 1/6 and 1 rotation period, respectively). The standard deviation of the distribution of span lengths in each case is 0.25 times the mean, as it has been in most of the previous studies. As before, we have also studied the case with no rotation breaks during a batch interval. As in TM98-02, the cases with rotation breaks all have the same degree of complexity in the rotation model: for each span, there are 5 ϕ , 2 α, and 2 δ coefficients. The runs with no rotation breaks include two different levels of model complexity: some with 95 ϕ coefficients in the (single) span and some with 24. We have used two different batch intervals in this study: both the 12 hr period (the same as in the two previous studies) and a 3 hr period (scaled to give the same number of rotations as in the previous studies).

Where possible, as in the previous studies, runs with rotation breaks are repeated with 16 different randomly chosen sets of span lengths, and the results are averaged to suppress the statistical noise from the span lengths. However, one set of runs (12 hr batch interval with 5 min average rotation spans) requires so much computing time for each case that we have dispensed with the 16-case averaging for that set. The need for averaging is reduced in these runs because there are four times as many spans in a batch as before, and we have verified (for a single value of the basic angle) that the difference between the 16-case average and the single case is acceptably small. The statistics of the 16 individual cases show a mean of -2.617 for the log of the uncertainty and a root-mean-square of 0.010 about that mean. For comparison, the one case used in preparing Figure 2 has a value of -2.631, only 1.4 times the RMS away from the mean.

As in TM98-02, the simulations come in sets with values of the basic angle in steps of 10 deg, from 0 to 180 deg, but with the first and last angles offset arbitrarily to 0.2 and 179.8 deg to avoid singularities in the simulation software.

III. RESULTS

The same figure of merit is used here as in the two previous memos. As before, the cohesion of the rotation model is gauged by an average of the uncertainty in $\Delta\phi$, the modeled difference in ϕ between pairs of epochs. See Section V of TM98-01 for the details. There is one new aspect in the present study, however. Instead of defining the grid of reference points at intervals of a fixed fraction of the batch interval, we have required them to have the same angular separation as before. Since the batch interval and rotation period were held fixed in the previous studies, this distinction was immaterial until now. Thus, the grid spacing of 0.01 times the batch interval stated in TM98-01 and TM98-02 was always an angular interval of 0.06 rotation (21.6 deg). In the present study, this angular spacing corresponds to 0.01 times the 3 hr batch intervals, but only 0.0025 times the 12 hr batch interval. In all runs presented here, the averaging has been done for lags of 11 through 50 of these units.

The main results of this study are presented in Figures 1 and 2. Each part of each figure shows a summary of a full set of runs at different basic angles. These results are qualitatively similar to the corresponding results presented in TM98-02. As before, the rotation model uncertainty goes through a broad minimum punctuated in some cases by "bad" angles. Figure 1 is especially similar to the corresponding figure in TM98-02, since the durations (spin period, batch interval, etc.) have all been scaled alike. The difference comes from the wider field of view in the present study, which results in more observations being made (about 41000 vs about 19000). Quantitatively, we expect the uncertainties to be smaller with more observations, and indeed they are. The larger number of observations would explain a descrease in the mean uncertainty by a factor of 1.47. The actual factors of decrease range from 1.80 to 2.04, suggesting an advantage from the cross linking. To make the comparisons plain, a summary of the results is presented in Table 1, much condensed by including only the values for basic angles of 100 deg (approximately the minimum of each curve in the figures) and 70 deg (a possible selection suggested by some aspects of the instrument's optical design). The results for 70 deg are slightly worse than for 100 deg in all cases, but never by more than 10%, and typically by much less.

In Table 1, the terms "high res." and "low res." refer to the resolution of the ϕ model for the single rotation span. The high-resolution cases have 16 times as many ϕ coefficients as there are rotations in the batch interval, while the low-resolution cases have only 4 times as many. Although it would have been preferable to use only one level of resolution throughout, the software has a limiting number of coefficients, which would be exceeded in the "high res." case with a batch interval longer than 6 rotations.

In the table, the description of spans as either "short" or "long" are also relative to the rotation period, "short" being 1/6 of a rotation (on average) and "long" being a whole rotation (on average). The table includes results both from this study (as shown in Figures 1 and 2) and from the previous study (as shown in Figure 1 of TM98-02).

IV. DISCUSSION

As in both previous studies, it remains clear that frequent rotation breaks should be avoided if at all possible. The worst single-span case shown in Table 1 (fewest observations, high resolution rotation model) gives a better result than the best case with multiple rotation breaks, even though the the latter represents almost ten times as many observations (albeit only about twice as many per revolution).

Interestingly, the improvement from the first column of results in Table 1 to the second is more than the simple root-N gain from the increased number of observations. In all three comparable cases, and for both basic angles tabulated, the improvement is about a factor of 2, almost as much as the increase in the number of observations. We take this to mean that the improvement is partly due to breaking degeneracy in the solutions (observing each star more times during the batch interval leads to a greater degree of interconnection of the rotation model). In contrast, the improvement from the second to the third column is much less than a root-N gain for the two cases with multiple rotation breaks. Between these two columns, the number of parameters to be

Table 1. Mean uncertainty in $\Delta\phi$ for two basic angles

Parameter	Values		
Display location	TM98-02	Figure 1	Figure 2
Field of view (deg)	0.75	1.60	1.60
Batch interval (hr)	12	3	12
Rotation period (min)	120	30	30
Batch interval (rot'ns)	6	6	24
Observations (1000)	19	41	165
Obs./rotation (1000)	3.2	6.9	6.9
Case	Results for 100 deg $\sigma(\Delta\phi)$ (mas)		
Short spans (6/rot'n)	1.005	0.537	0.370
Long spans (1/rot'n)	0.230	0.128	0.124
No breaks, high res.	0.092	0.045	
No breaks, low res.		0.036	0.017
Case	Results for 70 deg $\sigma(\Delta\phi)$ (mas)		
Short spans (6/rot'n)	1.059	0.586	0.389
Long spans (1/rot'n)	0.238	0.130	0.125
No breaks, high res.	0.096	0.047	
No breaks, low res.		0.037	0.018

estimated rises nearly (but not quite) in step with the number of observations, and, in some ways, the number of observations per rotation is a better measure of the available information than the total number of observations. It is therefore a little surprising that there is any improvement at all in the cohesion. However, the same mechanism can (and apparently does) operate to make the solutions more robust with a longer batch interval. Among other things, the third column includes a new phenomenon in that the total precession during a batch interval is no longer entirely negligible, being now 12 deg in the azimuthal angle α (corresponding to a change of more than 8 deg in the spacecraft axis direction).

The improvement from short spans to long spans is a factor ranging from 3 to 4.5 in these cases. This significant difference shows that, if the precession must be controlled by discrete events (rotation breaks), it would be advantageous to make those breaks as seldom as possible. In a pending memorandum, Reasenberg (1998) discusses the use of gas jets to precess the spacecraft and addresses a method of reducing the frequency of the gas jet firings. In the section on spacecraft operation, he introduces the use of two large firings separated by a short interval to do all the precessing needed for one rotation. Although there is a loss of some observing time and an increased use of ACS gas, Table 1 suggests that this approach should be considered.

V. REFERENCES

Chandler, J. F. and Reasenberg, R. D. 1998a, "GAMES sensitivity to complexity of rotation," TM98-01

Chandler, J. F. and Reasenberg, R. D. 1998b, "GAMES sensitivity II. Effect of the basic angle," TM98-02

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Appendix A. GLOSSARY

ACS: attitude control system.

Basic angle: the angle between the centers of the instrument's two fields of view. See also "opening angle."

Batch interval: the period during which data are collected for a single (first stage) analysis.

Cross-scan direction: the direction on the sky perpendicular to the direction of motion of the center of a field of view of the instrument; alternatively, the direction in the detector plane perpendicular to the time-averaged motion of a star image in the center of the field of view. (The center of the detector area may actually be obscured, but it remains the logical reference point.) More generally, the concept may be extended to refer to the un-averaged, instantaneous motion or the motion of a point away from the center. The direction local to a particular point in the field may differ from that of the center because of optical distortion.

Observing spiral: the path on the celestial sphere of the center of one of the instrument's fields of view during a batch interval. The two observing spirals are conceived to be very nearly coincident, except at the non-overlapping ends.

Observing spiral band: the region of the celestial sphere covered by one instrument field of view during a batch interval. Since the two spirals are nearly coincident, this term may also refer to the region of sky covered by both fields of view.

Opening angle: the angle between the centers of the instrument's two fields of view. See also "basic angle," the term used by the HIPPARCOS team.

Prereduction: a technique of speeding up least-squares parameter estimation by eliminating the "uninteresting" parameters from the normal equations. To within the numerical accuracy of computing, this technique gives exactly the same results (estimates and covariances) for the interesting parameters as would be obtained from a complete solution.

Rotation break: an attitude control event.

Rotation span: a period during which the spacecraft rotates without attitude control events. (The rotation spans are separated by attitude control events.)

Scan direction: the direction on the sky of the motion of the center of a field of view of the instrument; alternatively, the direction in the detector plane of the time-averaged motion of a star image at the center of the field of view. See the discussion of variants under "Cross-scan direction."

FIGURE CAPTIONS

Figure 1. (a) Geometric mean uncertainty in the difference in ϕ between pairs of points on the evenly-spaced grid, averaged over all lags from 11 to 50 times the grid spacing of 21.6 deg. Each point represents an average of 16 runs, each with an independent set of Gaussian-distributed rotation spans of 5 min average length and 1.25 min standard deviation. Each span has a separate rotation model consisting of 5 coefficients for ϕ, 2 for α, and 2 for δ. The batch interval is 3 hr.

(b) Same as (a), except that the distribution of rotation spans has a mean of 30 min and a standard deviation of 7.5 min.

(c) Same as (b), but plotted on an expanded scale.

(d) Similar to (a), but each point represents a single run with no rotation breaks. The overall rotation model has 95 coefficients for ϕ, 2 for α, and 2 for δ.

(e) Same as (d), except that the overall rotation model has 24 coefficients for ϕ (4 coefficients per spacecraft rotation).

Figure 2. (a) Same as 1a, except that the batch interval is 12 hr, and each point represents only one run.

(b) Same as (1b), except that the batch interval is 12 hr.

(c) Same as (b), but plotted on an expanded scale.

(d) Same as (1d), except that the batch interval is 12 hr. This is qualitatively similar to (1e), in appearance and in that there are only 4 ϕ coefficients per spacecraft rotation for both.

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