GREAT-CIRCLE VS. SPIRAL REDUCTION

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1 Introduction

Over the last several meetings, some people have said they do not understand the differences between the USNO and CfA data analysis plans. Others have incorrectly said the approaches are the same, and therefore one can be abandoned without harm to the project. These two statements, and the addition of new USNO people to the FAME MO&DA structure, necessitates the following brief paper where some of the differences between the two approaches are pointed out. The aspects of data analysis that are discussed here relate to the differing methods of spacecraft attitude determination, and differences in the conversion of image centers to positions on the sky. Other differences exist with regard to image centering and PSF fitting. It is felt, however, that the data analysis pipeline will be sufficiently modular that issues surrounding image centering may be dealt with in isolation from aspects of the data analysis that are further *down stream*.

The USNO approach to the post image-centering data analysis is to perform a *great-circle* reduction, similar to the Hipparcos analysis. The great-circle reduction is specifically designed to suppress zonal systematics, and results in a highly rigid, internally consistent set of one-dimensional positions. These positions, referred to as *abscissae*, are given in an arbitrary but inertial coordinate system called the *reference great circle*. Along-the-scan spacecraft attitude parameters are eliminated in the reduction, so the zero point of the abscissae is undetermined. The reference great circles are oriented and linked together in a separate *sphere-reconstruction* step using a subset of bright, single *tie stars*. The final step, following Lindegren's three-step approach, is to compute the astrometric parameters for all program stars.

The CfA plan, in contrast, is to perform what they term a *spiral* reduction, where the spacecraft attitude is determined for a batch of data using *a-priori* positions of a set of reference stars. These spirals, each consisting of a few to a few dozen individual spacecraft rotations, are then linked together to find the spacecraft attitude as a function of time over the entire mission. Since the attitude is determined in an absolute coordinate system, stellar positions can be tied directly to points on the celestial sphere as the last step. A major concern that we have with this approach is that zonal systematics from the input catalog may propagate into the final output catalog. It is also unclear that the global astrometric solution will be as rigid as one produced by a Hipparcos-style reduction.

Computational difficulties also arise in the CfA approach owing to their choice of coordinate system, and their attitude parameterization. The absolute coordinate system, in particular, seems to produce increased sensitivity to thruster usage owing to the strong coupling between cross-scan and along-scan attitude angles. The great-circle coordinates, on the other hand, are defined so that the cross-scan angles are always small, and have only a second-order effect on the abscissae. Thus, while thruster firings may degrade knowledge of the cross-scan angles, ten-times more frequent use of thrusters may be tolerable in the great-circle reduction.

The remainder of this note will elaborate on the great-circle reduction method as applied to FAME data. A description of the Hipparcos great-circle reduction is included for comparison purposes. Lastly, the great-circle and spiral reductions are contrasted.

2 Great-Circle Reduction

2.1 The Hipparcos Implementation

The Hipparcos consortia organized their observations into batches of five rotations spanning about ten hours. The plane in inertial space cutting through the center of such a data batch was termed a *reference great circle*, and served to define a convenient yet arbitrary inertial coordinate system. Observations were divided into *frames*, consisting of stars that were simultaneously in one or the other field of view. A new frame was formed after some number of stars had passed out of the FOV, and a similar number of new stars had entered, so that there was a significant overlap between adjacent frames. The basic observable for Hipparcos was the angular separation between stars within a frame. In fact, the quantities entering their equations of condition were the image locations relative to the average location of all the images in the frame, and projected onto the reference great circle.

In taking the difference between the image location and the average frame center, the *along-the-scan* attitude terms dropped out. The cross-scan attitude angles were still needed, however, to make second-order corrections to the one-dimensional abscissae. These angles, as well as the cross-scan position of the star, coupled only weakly to the along-the-scan equations. Thus, initial values for the cross-scan elements were assumed for a provisional great-circle reduction, and the procedure was iterated after better estimates of cross-scan elements became available.

2.2 FAME Implementation

As in the Hipparcos analysis, cross-scan attitude must be determined in order to properly compute the projection onto the reference great circle. Hipparcos used the Tycho star mapper data, combined with *a-priori* star positions to accomplish this. The cross-scan attitude was improved after an initial global solution, when the *a-priori* star positions were updated. FAME could proceed in much the same way, but using the cross-scan image locations on the CCD and *a-priori* positions. As an alternative, we can take advantage of our multiple, independent observations of each star per *visit*, and solve only for *changes* in the cross-scan angles. This works because attitude in an absolute coordinate system is not needed in the great-circle reduction, and has the advantage of eliminating *a-priori* positions from the condition equations.

Along-the-scan attitude information is also required for both missions. For FAME, along-scan attitude knowledge is only required to be good enough to cut "postage stamps". Hipparcos had the additional requirement to resolve *grid-step ambiguities*, which were a feature of its modulation grid/image dissector detection system. Other than these relatively loose requirements – which are in actuality placed on the input catalog – there is NO need for along-the-scan attitude for either mission, because this attitude component *drops out* when *relative* image locations along the reference great circle are formed. For FAME, there is an additional complication because relative angles are not directly measured as they are for Hipparcos. Rather, the time of transit of a stellar image across a particular CCD is recorded. The relative angular separations of a few hundred stars, transiting within a few seconds of one another, can be obtained if the angular velocity about the spin axis is estimated. In this case, the analysis can proceed virtually identically to the Hipparcos analysis. This approach would be the most stable with regard to thruster firings or micrometeorites, and would involve the least risk owing to the demonstrated success of the Hipparcos mission.

A second approach to the great-circle reduction for FAME is to find the difference in along-scan attitude from multiple detections of the same star at different times. Again, the *absolute* attitude is not important, and is not determined. Once the relative along-scan attitude as a function of time is known, abscissae are easily obtained from the time stamps of the observations. This approach has been simulated in the case of zero cross-scan angles, and shown to work. It is somewhat less stable with regard to thruster firings, requiring a significant fraction of a complete rotation in between thruster events. However, this approach may be the best for those scans containing an Earth or Moon transit.

3 Comparison with Spiral Reduction

The Euler angles used in the spiral reduction are the angle between the spin axis and the Sun vector, the azimuth of the spin axis about the Sun vector, and the rotation angle about the spin axis. These angles are all large, and no small-angle approximations are permissible. It has been shown by Chandler and Reasenberg that the necessity of solving for the Sun angle and the azimuth of precession lead to sensitivity with respect to thruster firings, and the resulting requirement to fire thrusters no more than about once per day. The great-circle analysis is significantly more robust because angles are determined only relative to the current mean plane of observations, ie. reference great circle. Thus, uncertainty in cross-scan angles has a small effect on the accuracy of the abscissae. Moreover, the loose coupling between cross-scan and along-scan equations in the great circle approach allows an iterative solution method, where cross-scan and along-scan equations are solved separately.

In the CfA analysis, the spacecraft attitude is computed as a function of time over the whole mission. The star positions are obtained afterwards by back substitution. Since the attitude is required to be in an absolute coordinate system, valid for all scans throughout the mission, the *a-priori* positions of reference stars in the same coordinate system play an essential role. Corrections to the reference star positions *could* be computed simultaneously with the spacecraft attitude, but they are in fact discarded for computational efficiency. It is yet to be demonstrated that zonal systematic errors in the reference star catalog can be prevented from propagating through to the output catalog. In the great-circle analysis, *a-priori* positions are not used for attitude determination. The attitude is determined in an arbitrary coordinate system, and the attitude parameters are in fact discarded. The attitude in an absolute coordinate system is never computed, and the along-scan attitude actually drops out.

The CfA attitude determination requires a complex model of spacecraft dynamics, including knowledge of all external torques. Many of these torques, such as those due to radiation pressure and gravity gradients, will be periodic at harmonics of the spacecraft rotational frequency. Thus, they will not all be linearly independent. Moreover, it is impossible to include all torques in the model, and a set of utility parameters must be included to mop up unmodeled effects. As a fall back plan, CfA will construct a Kalmann filter for attitude estimation.

The great-circle reduction requires only a kinematic description of spacecraft motion. That is, we do not care *why* the motion is what it is, only that we can model it. This modeling will be done with a set of orthogonal functions which avoids degeneracy in the condition equations. A Fourier series representation of angular velocity about the spin axis has provisionally been adopted, which transfers position errors from the spatial domain to the spatial-frequency domain. Thus, while local correlations may exist in positional errors, the errors are more or less uniformly distributed around the reference great circle, preventing large-scale zonal errors.

Overall, the great-circle reduction is computationally efficient, since the solution may be performed iteratively, and attitude parameters are not estimated. In the spiral reduction, corrections to the *a-priori* reference star positions enter the condition equations, and seem to require estimation along with the attitude parameters. This computational burden is avoided by what is termed *partial prereduction*, whereby the position corrections are eliminated. The computational load of the Kalmann filter is TBD.

Once the one-dimensional abscissae have been obtained, the individual reference great circles must be tied together to reconstruct the celestial sphere. This complex problem has been solved already, allowing FAME to leverage off of the Hipparcos heritage. In the spiral reduction, the individual spirals must also be tied together. This procedure is essentially new, but in principle straight forward.